Linear Algebra, Winter 2022

# List 4

*Determinants, inverses, systems*

74. (a) Calculate $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^2 = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix} = \boxed{\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}}$

(b) Calculate $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^4 = \boxed{\begin{bmatrix}5 & 3\\3 & 2\end{bmatrix}}$

(b) Calculate $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^5 = \boxed{\begin{bmatrix}8 & 5\\5 & 3\end{bmatrix}}$

(c) Calculate $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^{10} = \boxed{\begin{bmatrix}89 & 55\\55 & 34\end{bmatrix}}$

☆(d) Looking at $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^2$, $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^3$, etc., guess a formula for $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^n$.

For any $n \geq 1$, we have $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}^n = \begin{bmatrix}F_{n+1} & F_n\\F_n & F_{n-1}\end{bmatrix}$, where $F_n$ is the $n^{\text{th}}$ Fibonacci number ($F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$).

Direct formula: $\frac{1}{\sqrt{5}}\begin{bmatrix}\phi^{n+1} - \psi^{n+1} & \phi^n - \psi^n\\ \phi^n - \psi^n & \phi^{n-1} - \psi^{n-1}\end{bmatrix}$, where $\phi = \frac{1+\sqrt{5}}{2}$ and $\psi = \frac{1-\sqrt{5}}{2}$.

75. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be $T(x,y) = (2x+y, -x+2y)$ and let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be $S(x,y) = (3y, -x)$. Compute $S(T(3,1))$. $\boxed{(-3,-7)}$

76. For $T$ and $S$ from Problem 75, give the matrix for $T$, the matrix for $S$, and matrix for $S(T(x,y))$.

$M_T = \begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}$, $M_S = \begin{bmatrix}0 & 3\\-1 & 0\end{bmatrix}$, $M_S M_T = \boxed{\begin{bmatrix}-3 & 6\\-2 & -1\end{bmatrix}}$

77. (a) Is $\begin{bmatrix}2 & 5\\9 & -2\end{bmatrix}\begin{bmatrix}4\\4\end{bmatrix}$ parallel to $\begin{bmatrix}4\\4\end{bmatrix}$? $\boxed{\text{Yes}}$

(b) Is $\begin{bmatrix}2 & 5\\9 & -2\end{bmatrix}\begin{bmatrix}2\\5\end{bmatrix}$ parallel to $\begin{bmatrix}2\\5\end{bmatrix}$? $\boxed{\text{No}}$

(c) Is $\begin{bmatrix}2 & 5\\9 & -2\end{bmatrix}\begin{bmatrix}2\\9\end{bmatrix}$ parallel to $\begin{bmatrix}2\\9\end{bmatrix}$? $\boxed{\text{No}}$

(d) Is $\begin{bmatrix}2 & 5\\9 & -2\end{bmatrix}\begin{bmatrix}5\\-9\end{bmatrix}$ parallel to $\begin{bmatrix}5\\-9\end{bmatrix}$? $\boxed{\text{Yes}}$

(e) Is $\begin{bmatrix}2 & 5\\9 & -2\end{bmatrix}\begin{bmatrix}2\\-9\end{bmatrix}$ parallel to $\begin{bmatrix}2\\-9\end{bmatrix}$? $\boxed{\text{No}}$

(f) Is $\begin{bmatrix}2 & 5\\9 & -2\end{bmatrix}\begin{bmatrix}2\\-2\end{bmatrix}$ parallel to $\begin{bmatrix}2\\-2\end{bmatrix}$? $\boxed{\text{No}}$

The **determinant** of a square matrix $A$ is written as $\det(A)$. For a $2 \times 2$ matrix,

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc.$$

For larger matrices, the formula is more difficult. Properties include

$$\det(AB) = \det(A) \cdot \det(B) \qquad \text{and} \qquad \det(sA) = s^n \det(A)$$

if $A$ is an $n \times n$ matrix. Geometrically, $|\det(A)|$ is the volume of the parallelepiped (in 2D, area of the parallelogram) whose edges, as vectors, are the columns of $A$.

78. Compute the determinants of the following matrices.

(a) $\det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \boxed{-2}$

(b) $\det\left(\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}\right) = \boxed{2}$

(c) $\det\left(\begin{bmatrix} 8 & 4 \\ 6 & 3 \end{bmatrix}\right) = \boxed{0}$

(d) $\det\left(\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}\right) = \boxed{17}$

(e) $\det\left(\begin{bmatrix} 3 & b \\ 2 & 5 \end{bmatrix}\right) = \boxed{15 - 2b}$

(f) $\det\left(\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 9 \\ 5 & 6 & 8 \end{bmatrix}\right) = \boxed{25}$

☆(g) $\det\begin{bmatrix} 3 & 1 & 3 & 3 \\ 1 & 2 & 4 & 0 \\ 2 & 0 & 2 & 2 \\ 4 & 2 & 1 & 3 \end{bmatrix} = \boxed{-12}$

79. If $M$ is a $5 \times 5$ matrix with $\det(M) = 2$ compute $\det(2M) = 2^5 \cdot 2 = \boxed{64}$ and $\det(-3M^2) = (-3)^5 \cdot 2 \cdot 2 = \boxed{-972}$

The $\boldsymbol{n \times n}$ **identity matrix** is the matrix $I$ (also written $I_n$ or $I_{n \times n}$) such that

$$IM = MI = M$$

for any $n \times n$ matrix $M$. It has 1 along the main diagonal and 0 everywhere else.

80. (a) Multiply $\begin{bmatrix} 0 & -4 & 2 \\ 10 & -1 & 0 \\ 1 & 12 & -6 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 30 & -1 & 10 \\ \frac{121}{2} & -2 & 20 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

(b) Multiply $\begin{bmatrix} 0 & -4 & 2 \\ 10 & -1 & 0 \\ 1 & 12 & -6 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 30 & -1 & 10 \\ \frac{121}{2} & -2 & 20 \end{bmatrix} \begin{bmatrix} 6 & 4 & 9 \\ 0 & -3 & -2 \\ 4 & 6 & -4 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 4 & 9 \\ 0 & -3 & -2 \\ 4 & 6 & -4 \end{bmatrix}}$

The **inverse matrix** of a square matrix $M$ is written $M^{-1}$ (spoken as "M inverse") and it is the unique matrix for which $M^{-1}\,M = I$. An inverse matrix exists if and only if $\det(M) \neq 0$. For a $2 \times 2$ matrix,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \tfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

For larger matrices, the formula is much more difficult but includes $\frac{1}{\det(M)}$.

81. Find $\begin{bmatrix} 5 & 4 \\ 1 & -2 \end{bmatrix}^{-1} = \boxed{\begin{bmatrix} \frac{1}{7} & \frac{2}{7} \\ \frac{1}{14} & -\frac{5}{14} \end{bmatrix}}$ and $\begin{bmatrix} 0 & -4 & 2 \\ 10 & -1 & 0 \\ 1 & 12 & -6 \end{bmatrix}^{-1} = \boxed{\begin{bmatrix} 3 & 0 & 1 \\ 30 & -1 & 10 \\ \frac{121}{2} & -2 & 20 \end{bmatrix}}$ from Task 80(a).

82. Find the inverses of the matrices from Task 78, if they exist. (☆(f), ☆(g))

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{9}{2} & -\frac{5}{2} \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 4 \\ 6 & 3 \end{bmatrix}^{-1}$ does not exist

(d) $\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{5}{17} & \frac{1}{17} \\ -\frac{2}{17} & \frac{3}{17} \end{bmatrix}$

(e) $\begin{bmatrix} 3 & b \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{5}{15-2b} & -\frac{b}{15-2b} \\ -\frac{2}{15-2b} & \frac{3}{15-2b} \end{bmatrix}$

☆(f) $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 9 \\ 5 & 6 & 8 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{22}{25} & \frac{6}{25} & \frac{7}{25} \\ \frac{29}{25} & -\frac{17}{25} & \frac{1}{25} \\ -\frac{8}{25} & \frac{9}{25} & -\frac{2}{25} \end{bmatrix}$

(g) $\begin{bmatrix} 3 & 1 & 3 & 3 \\ 1 & 2 & 4 & 0 \\ 2 & 0 & 2 & 2 \\ 4 & 2 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & \frac{1}{3} & 2 & \frac{2}{3} \\ 1 & 0 & -\frac{3}{2} & 0 \\ 0 & \frac{1}{6} & \frac{1}{4} & -\frac{1}{6} \\ 2 & -\frac{1}{2} & -\frac{7}{4} & -\frac{1}{2} \end{bmatrix}$

83. Find the matrix $M$ from Task 59. $\boxed{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 5 & 3 \end{bmatrix}}$

84. For each of the following, does an inverse matrix exist?

(a) Matrix $A$, a $3 \times 3$ matrix with $\det(A) = 3$.

(b) Matrix $B$, a $3 \times 5$ matrix where every number in the matrix is 1.

(c) Matrix $C$, a $4 \times 4$ matrix where every number in the matrix is 1.

(d) Matrix $D$, a $4 \times 4$ matrix where every number in the matrix is 0.

(e) Matrix $E$, a $5 \times 5$ matrix with $\det(D) = -1$.

(f) Matrix $F$, a $7 \times 7$ matrix with $\det(E) = 0$.

(g) Matrix $G$, a $2 \times 2$ matrix with $a_{ij} = i + j$.

Only $A$ and $E$ and $G$ have an inverse

85. For what values of $p$ are each of the following matrices invertible? Give a formula for the inverse of each matrix.

(a) $\begin{bmatrix} 1 & 2 \\ p & p^3 \end{bmatrix}$ If $p \neq 0, p \neq \sqrt{2}, p \neq -\sqrt{2}$, inverse is $\frac{1}{p^3-2p}\begin{bmatrix} p^3 & -2 \\ -p & 1 \end{bmatrix}$.

(b) $\begin{bmatrix} \cos p & -\sin p \\ \sin p & \cos p \end{bmatrix}$ Inverse is $\begin{bmatrix} \cos p & \sin p \\ -\sin p & \cos p \end{bmatrix}$ (this is **clockwise** rotation by $p$).

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - pI_{2\times 2}$ If $p \neq 4, p \neq -1$, inverse is $\frac{1}{p^2-3p-4}\begin{bmatrix} 2-p & -2 \\ -3 & 1-p \end{bmatrix}$.

86. Let $f : \mathbb{R}^2 \to R^2$ be given by $f(x, y) = (9x - 11y, 4x - 5y)$. Find values of $a$ and $b$ such that $f(a, b) = (7, 2)$. $(a, b) = (13, 10)$

87. Let $f : \mathbb{R}^2 \to R^2$ be given by $f(x, y) = (5x, 10x + y)$. Give a formula for $f^{-1}(x, y)$, that is, the function for which $f^{-1}(f(x, y)) = (x, y)$. $(\frac{x}{5}, -2x + y)$

A collection of vectors is called **linearly dependent** if one of the vectors is a linear combination of the others. Otherwise it is **linearly independent**.

Alternate definition: a collection $\{\vec{v_1}, ..., \vec{v_n}\}$ is linearly dependent if there exist scalars $s_1, ..., s_n$ not *all* zero (but some may be zero) such that $s_1\vec{v_1} + \cdots + s_n\vec{v_n} = \vec{0}$.

88. For the vectors

$$\vec{v_1} = [2, 9, -6], \qquad \vec{v_2} = [4, 2, -6], \qquad \vec{v_3} = [0, -8, 3],$$

is the collection $\{\vec{v_1},\ \vec{v_2},\ \vec{v_3}\}$ linearly dependent or independent?
Hint: See Task 28 from List 1.

Because $\vec{v_1} = \frac{1}{2}\vec{v_2} + (-1)\vec{v_3}$, the collection is linearly dependent.

89. Determine whether each collection is linearly dependent or independent.

(a) $\{\hat{\imath},\ \hat{\jmath}\}$ linearly <u>in</u>dependent

(b) $\{\hat{\imath},\ \hat{\jmath},\ \hat{k}\}$ linearly <u>in</u>dependent

(c) $\{\hat{\imath},\ \hat{\jmath},\ \vec{0}\}$ linearly dependent

(d) $\left\{ \begin{bmatrix} 44 \\ 1 \end{bmatrix}, \begin{bmatrix} -9 \\ 21 \end{bmatrix}, \begin{bmatrix} 21 \\ 49 \end{bmatrix}, \begin{bmatrix} 8 \\ 8 \end{bmatrix} \right\}$ linearly dependent

(e) $\left\{ \begin{bmatrix} 15 \\ -35 \end{bmatrix} \right\}$ linearly <u>in</u>dependent

(f) $\left\{ \begin{bmatrix} 15 \\ -35 \end{bmatrix}, \begin{bmatrix} -9 \\ 21 \end{bmatrix} \right\}$ linearly dependent

(g) $\left\{ \begin{bmatrix} 5 \\ 50 \\ -100 \end{bmatrix}, \begin{bmatrix} 0 \\ 100 \\ 200 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ linearly independent

90. For each, state whether the collection must be linear dependent, must be linear independent, or that there is not enough information to know this.

    (a) A collection of 10 vectors each of dimension 3. dependent

    (b) A collection of 3 vectors each of dimension 10. not enough info

    (c) A collection of 20 vectors each of dimension 10. dependent

    (d) A collection of 3 vectors, each of dimension 3, that includes two parallel vectors. dependent

    (e) A collection of 3 vectors, each of dimension 3, that includes two perpendicular vectors. not enough info

    (f) A collection of 3 vectors, each of dimension 3, where each vector is perpendicular to the other two. independent

    (g) A collection of 3 vectors, each of dimension 3, that includes the zero vector. dependent

91. Is the collection of vectors

$$\left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

linearly dependent or linearly independent? dependent

92. Give an example of a vector $\vec{u}$ for which

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \vec{u} \right\}$$

is linearly independent. Any non-zero vector $[a, b, c]$ that does not have $c = -2b$ will work. The simplest example is $[0, 0, 1]$.

93. Write $[8, 3, 1]$ as a linear combination of $[1, 0, 1]$ and $[2, 0, 3]$ or explain why it is impossible to do so. Impossible because $a[1, 0, 1] + b[2, 0, 3] = [a + 2b, 0, 1 + 3b]$ will always have 0 has the second component, but $[8, 3, 1]$ as a 3 as the second component.

94. Write $[5, 5, 1]$ as a linear combination of $[1, 1, 1]$ and $[0, 0, 8]$ or explain why it is impossible to do so. $[5, 5, 1] = 5[1, 1, 1] + (-\frac{1}{2})[0, 0, 8]$.

95. (a) Is the collection $\{[-1, 8, 8]\}$ linearly independent? Yes

    (b) Is the collection $\{[-1, 8, 8], [5, 0, 0]\}$ linearly independent? Yes

    (c) Is the collection $\{[-1, 8, 8], [5, 0, 0], [3, 1, 3]\}$ linearly independent? Yes

    (d) Is the collection $\{[-1, 8, 8], [5, 0, 0], [3, 1, 3], [3, -4, 4]\}$ linearly independent? No

96. (a) Is the collection $\{[0, 2, 5]\}$ linearly independent? Yes

(b) Is the collection $\{[0,2,5],[1,1,-4]\}$ linearly independent? Yes

(c) Is the collection $\{[0,2,5],[1,1,-4],[2,4,-3]\}$ linearly independent? No

(d) Is the collection $\{[0,2,5],[1,1,-4],[2,4,-3],[2,8,7]\}$ linearly independent? No

The **rank** of a matrix is the maximum number of linearly independent rows in the matrix. This is also the maximum number of linearly independent columns.

97. Give the rank of the following matrices: (a) 1, (b) 2, (c) 3, (d) 3

(a) $\begin{bmatrix} -1 & 8 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 8 & 8 \\ 5 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 8 & 8 \\ 5 & 0 & 0 \\ 3 & 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 8 & 8 \\ 5 & 0 & 0 \\ 3 & 1 & 3 \\ 3 & -4 & 4 \end{bmatrix}$

98. Give the rank of the following matrices: (a) 1, (b) 2, (c) 2, (d) 2

(a) $\begin{bmatrix} 0 & 2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 & 5 \\ 1 & 1 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & 5 \\ 1 & 1 & -4 \\ 2 & 4 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2 & 5 \\ 1 & 1 & -4 \\ 2 & 4 & -3 \\ 2 & 8 & 7 \end{bmatrix}$

99. Explain why $\text{rank}(M) = 3$ for the matrix $M = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 3 & 5 \\ 6 & 3 & 2 \end{bmatrix}$. The column $\begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}$ cannot be a linear combination of the other two because they both have a 0 in the first entry.

The column $\begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$ cannot be a linear com. of only $\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$ because all multiples of $\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$ have the same $y$- and $z$-component. Therefore all 3 columns are linearly independent, so the rank is 3.

Alternatively, $\det(M) = 4\det\left(\begin{smallmatrix} 3 & 5 \\ 3 & 2 \end{smallmatrix}\right) - 0 + 0 = 4(6-15) \neq 0$, and the rank of a $3 \times 3$ matrix is less than 3 if and only if its determinant is 0.

☆100. For which values of $p$ does $\begin{bmatrix} 4 & 0 & 0 \\ 1 & 3 & 5 \\ 6 & 3 & p \end{bmatrix}$ have rank 2? $p = 5$

For which values does it have rank 3? all $p \neq 5$ Rank 1? None

101. Find $\text{rank}(A)$ and $\det(A)$ for the matrix $A = \begin{bmatrix} 4 & 8 & 1 \\ 7 & 14 & 6 \\ 1 & 2 & 6 \end{bmatrix}$ without a calculator.

$\text{rank}(A) = 2$ because the second column is two times the first column.

$\det(A) = 0$ because the rank is less than the number of columns.

102. The rank of $\left[\begin{smallmatrix} 4 & 1 & 5 \\ 2 & 0 & 2 \\ 1 & -1 & 0 \end{smallmatrix}\right]$ is 2 and the rank of $\left[\begin{smallmatrix} 4 & 1 & 5 & 1 \\ 2 & 0 & 2 & 2 \\ 1 & -1 & 0 & 7 \end{smallmatrix}\right]$ is 3.

How many solutions does the system $\begin{cases} 4x + y + 5z = 1 \\ 2x + \phantom{y+{}} 2z = 2 \\ x - y \phantom{+5z} = 7 \end{cases}$ have? None

103. The rank of $\left[\begin{smallmatrix} 4 & 1 & 5 \\ 2 & 0 & 2 \\ 1 & -1 & 0 \end{smallmatrix}\right]$ is 2 and the rank of $\left[\begin{smallmatrix} 4 & 1 & 5 & 6 \\ 2 & 0 & 2 & 4 \\ 1 & -1 & 0 & 4 \end{smallmatrix}\right]$ is also 2.

How many solutions does the system $\begin{cases} 4x + y + 5z = 6 \\ 2x + \phantom{y+{}} 2z = 4 \\ x - y \phantom{+5z} = 4 \end{cases}$ have? Infinitely many

104. The rank of $\left[\begin{smallmatrix} 7 & 2 & 5 \\ 9 & 0 & 3 \\ 3 & -1 & 0 \end{smallmatrix}\right]$ is 3 and the rank of $\left[\begin{smallmatrix} 7 & 2 & 5 & -1 \\ 9 & 0 & 3 & 0 \\ 3 & -1 & 0 & 5 \end{smallmatrix}\right]$ is also 3.

How many solutions does the system $\begin{cases} 7x + 2y + 5z = -1 \\ 9x + \phantom{2y+{}} 3z = 0 \\ 3x - \phantom{2}y \phantom{+5z} = 5 \end{cases}$ have? One

☆105. If the numbers $a, b, c, d$ are such that $(x, y) = (9, 1)$ is a solution to $\begin{cases} ax + by = 2 \\ cx + dy = 3 \end{cases}$ but the system $\begin{cases} ax + by = 4 \\ cx + dy = 9 \end{cases}$ has no solutions, what is the rank of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$?

Since the system $\begin{cases} ax + by = 4 \\ cx + dy = 9 \end{cases}$ has no solutions, the rank is $< 2$. Because $\begin{cases} ax + by = 2 \\ cx + dy = 3 \end{cases}$ has at least one solution, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is not $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and thus the rank is $> 0$. The only integer strictly between 0 and 2 is 1.
(The values $2, 3, 4, 9$ are not important.)