Linear Algebra, Winter 2022

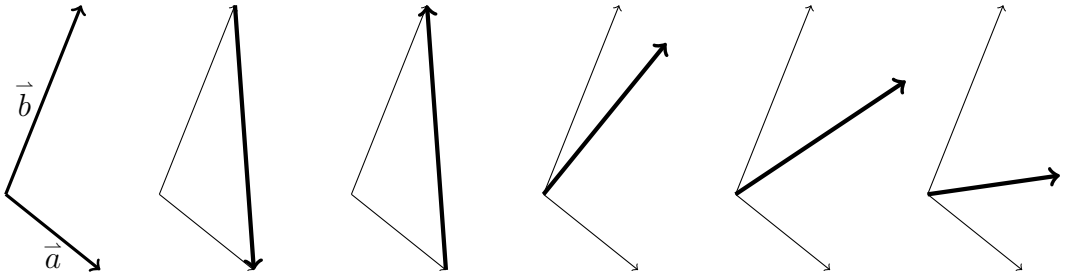
List 5

Review for Celebration of Knowledge 1

106. If $\vec{a} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ and $\vec{b} = [1, 0, -1]$ and $\vec{c} = 12\hat{k}$, calculate $\vec{a} + 4\vec{b} - \frac{1}{2}\vec{c}$.

107. Calculate $(9\hat{\imath} + 4\hat{k}) \cdot (5\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and $(9\hat{\imath} + 4\hat{k}) \times (5\hat{\imath} - \hat{\jmath} + 2\hat{k})$.

108. Setup Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

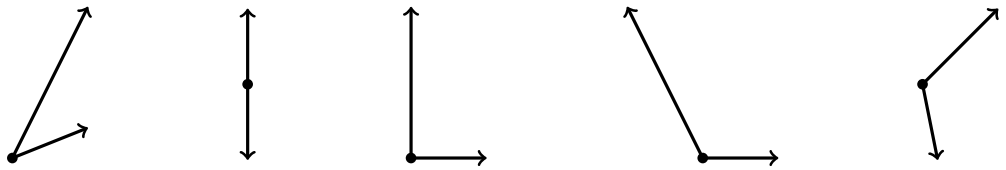


Which picture shows $\vec{a} + \vec{b}$? Which shows $\vec{a} - \vec{b}$?

109. Find the cosine of the angle between $10\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} + 10\hat{\jmath}$.

☆110. If $\vec{a}$ and $\vec{b}$ point in the same direction, $4\vec{c}$ and $8\vec{b}$ have the same length, and $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, find the angle between $\vec{a}$ and $\vec{c}$.

111. Which picture(s) below have $\vec{u} \cdot \vec{v} = 0$? Which have $\vec{u} \cdot \vec{v} > 0$?



(a) (b) (c) (d) (e)

☆112. If $A = (0, 0)$ and $B = (4, 3)$, find all possible positions for the point C such that ABC is a right isosceles triangle (that is, two of its sides have the same length).

113. Write $18\hat{\imath} + \hat{\jmath}$ as a linear combination of $\vec{v} = \hat{\imath} + 2\hat{\jmath}$ and $\vec{w} = 2\hat{\imath} - 3\hat{\jmath}$.

114. Write $\begin{bmatrix} -20 \\ 12 \\ -32 \end{bmatrix}$ as a linear combination of $\vec{a} = \begin{bmatrix} 15 \\ -9 \\ 24 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 10 \\ 12 \\ -8 \end{bmatrix}$.

115. Write $\begin{bmatrix} 17 \\ -13 \\ 63 \end{bmatrix}$ as a linear combination of $\vec{u} = \begin{bmatrix} 9 \\ 1 \\ 25 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

116. Are the vectors[1] $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 10 \\ -4 \end{bmatrix}$ linear dependent or linear independent?

[1] Technically, this should ask whether the *collection* (or *set*) of vectors $\{[5, 2],\ [10, -4]\}$ is a linearly dependent collection or a linearly independent collection. But it is common to say that "$\vec{u}$ and $\vec{v}$ are linearly (in)dependent" when the set $\{\vec{u}, \vec{v}\}$ is linearly (in)dependent.

117. Are the vectors $\begin{bmatrix}5\\2\end{bmatrix}, \begin{bmatrix}10\\-4\end{bmatrix}, \begin{bmatrix}7\\3\end{bmatrix}$ linear dependent or linear independent?

118. Determine whether each of the following collections of vectors are linear independent or linearly dependent:

(a) $\{[6,2]\}$

(b) $\{[6,2], \quad [3,0]\}$

(c) $\{[6,2], \quad [3,0], \quad [0,1]\}$

(d) $\{[6,2], \quad [3,1]\}$

(e) $\{[6,2], \quad [3,1], \quad [0,1]\}$

(f) $\{[6,2], \quad [3,1], \quad [9,3]\}$

☆119. If $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent, determine whether each of the following collections of vectors are linear independent or linearly dependent:

(a) $\{\vec{u}, \vec{v}\}$ (b) $\{\vec{u}, \vec{v}, \vec{u}+\vec{v}\}$ (c) $\{\vec{u}, \vec{v}, \vec{u}+\vec{w}\}$ (d) $\{\vec{u}, \vec{v}, 3\vec{w}\}$

120. Which of the following lines is parallel to the line $\begin{cases} x = 9 + 8t \\ y = 11 - 6t \\ z = 1 + 10t \end{cases}$?

(A) $\begin{cases} x = 1 + 4t \\ y = -7 - 3t \\ z = 2 + 5t \end{cases}$ (B) $\begin{cases} x = 7 + 8t \\ y = 12 - 4t \\ z = 4t \end{cases}$ (C) $\begin{cases} x = 2 - 4t \\ y = 6 - 3t \\ z = 4 + 5t \end{cases}$ (D) $\begin{cases} x = 8 + 9t \\ y = -6 + 11t \\ z = 10 + t \end{cases}$

121. Which line from Task 120 is *parallel* to the plane

$$4(x-7) - 2(y-9) + 2(z+3) = 0?$$

122. Which line from Task 120 is *perpendicular* to the plane from Task 121?

123. Find the intersection of the line $\begin{cases} x = 1 + t \\ y = 2 - 2t \\ z = 8 - 5t \end{cases}$ and the plane $8x + 2y - z = 10$.

124. (a) Find the intersection of the lines

$$\begin{array}{llll} L_1: & x = 1 + 9t, & y = 13, & z = 7 + 4t \\ L_2: & x = 3 + 5s, & y = 18 - s, & z = 9 + 2s. \end{array}$$

(b) Find a vector that is perpendicular to both lines.

(c) Give an equation for the plane that contains L_1 and L_2.

125. What are the dimensions of $\begin{bmatrix}7 & \frac{1}{10}\end{bmatrix}\begin{bmatrix}\frac{1}{3} & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}6 & -33 & 2\\ 0 & 0 & 0\end{bmatrix}$?

126. Calculate the product in Task 125.

127. If $A = \begin{bmatrix}4 & 0 & 0 & -2 & -6\\ 1 & 1 & 1 & 1 & 1\\ 1 & 3 & 0 & 19 & -8\end{bmatrix} B$, and matrix A is invertible, what are the dimensions of matrix A and the dimension of matrix B?

128. Multiply the following matrices, or state that the product does not exist.

(a) $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 6 & 7\\ 8 & 9 & 10\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 6\\ 7 & 8\\ 9 & 10\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}5 & -1\\ 1 & 1\end{bmatrix}$

(d) $\begin{bmatrix}5 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$

(e) $\begin{bmatrix}1 & 0 & 2\\ 5 & 0 & 5\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 5 & 0 & 5\end{bmatrix}$

(f) $\begin{bmatrix}1 & 0 & 2\\ 5 & 0 & 5\end{bmatrix}\begin{bmatrix}4 & 2 & 0\\ 0 & 2 & 4\\ 1 & -1 & 3\end{bmatrix}$

(g) $\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}15 & 8 & -2\\ 3 & 5 & 1\\ 9 & 9 & 2\end{bmatrix}$

129. Which of the following are linear transformations?

(a) $f(x, y) = (x + 10, y)$

(b) $f(x, y) = (10x, y)$

(c) $f(x, y) = (x + 2y, x - 2y)$

(d) $f(x, y) = (x + 2y, y - 2x)$

(e) $f(x, y) = (100x^2, y)$

130. If $f(x, y) = (x + y, 0)$ and $g(x, y) = (5x - y, x + y)$, give a formula for $f(g(x, y))$ and a formula for $g(f(x, y))$.

131. Calculate the determinant and the inverse of $\begin{bmatrix}5 & 1\\ 8 & 2\end{bmatrix}$.

132. Calculate the determinant of $\begin{bmatrix}11 & 10 & 7\\ 1 & 0 & 0\\ 11 & 18 & 15\end{bmatrix}$.

133. If A is a 6×6 matrix with $\det(A) = 5$, and B is a 6×2 matrix, which of the following exist?

(a) $2A + B$ (b) $3B + A$ (c) AB (d) BA (e) $I_{6\times6} + A$ (f) $I_{6\times6} + B$ (g) $I_{6\times6}A$ (h) $I_{6\times6}B$ (i) A^{-1} (j) B^{-1} (k) $A^{-1} + B^{-1}$ (ℓ) $A^{-1}B$

134. Solve the following systems of equations, if they have solutions.

(a) $\begin{cases} x + 8y = 9 \\ x - 12y = -1 \end{cases}$ (b) $\begin{cases} 10x - 4y = 5 \\ 5x - 2y = 10 \end{cases}$ ☆(c) $\begin{cases} 10x - 4y = 10 \\ 5x - 2y = 5 \end{cases}$

135. Calculate the rank of $\begin{bmatrix}6 & 2\\ 3 & 0\\ 0 & 1\end{bmatrix}$ and the rank of $\begin{bmatrix}6 & 2\\ 3 & 1\\ 9 & 3\end{bmatrix}$.

136. Calculate the rank of $\begin{bmatrix}6 & 3 & 0\\ 2 & 0 & 1\end{bmatrix}$ and the rank of $\begin{bmatrix}6 & 3 & 9\\ 2 & 1 & 3\end{bmatrix}$.

137. The determinant of $\begin{bmatrix}-4 & 19 & -10 & 6\\ -10 & 19 & 19 & -5\\ 10 & 10 & 8 & -5\\ 2 & 7 & -12 & 5\end{bmatrix}$ is 36. What is its rank?