Linear Algebra, Winter 2022

List 6

Eigenvalues, complex number intro

Let M be a square matrix. If
$$M\vec{v} = \lambda\vec{v}$$
with $\vec{v} \neq \vec{0}$ then the vector $\vec{v}$ is called an **eigenvector** of M and the number λ is called an **eigenvalue** of M.
• The eigenvalues of M are exactly the numbers λ for which
$$\det(A - \lambda I) = 0.$$
• The determinant of M is exactly equal to the product of all its eigenvalues.

138. Find an eigenvector of $\begin{bmatrix} 7 & -5 \\ 0 & 8 \end{bmatrix}$ corresponding to the eigenvalue 8. That is, find a non-zero vector $\begin{bmatrix} x \\ y \end{bmatrix}$ such that $\begin{bmatrix} 7 & -5 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 8\begin{bmatrix} x \\ y \end{bmatrix}$.

139. Find the eigenvalues of $\begin{bmatrix} 4 & 1 \\ -2 & 8 \end{bmatrix}$.

140. (a) Find the eigenvalues of $\begin{bmatrix} 2 & 1 \\ 7 & 8 \end{bmatrix}$. (b) Find the eigenvectors of $\begin{bmatrix} 2 & 1 \\ 7 & 8 \end{bmatrix}$.

141. (a) Find the eigenvalues of $\begin{bmatrix} 4 & 1 \\ -8 & 8 \end{bmatrix}$. ☆(b) Find the eigenvectors of $\begin{bmatrix} 4 & 1 \\ -8 & 8 \end{bmatrix}$.

142. Find the eigenvalues of $\begin{bmatrix} 4 & 8 & 1 \\ 7 & 14 & 6 \\ 1 & 2 & 6 \end{bmatrix}$.

143. If the matrix M satisfies
$$M\begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \end{bmatrix} \quad\text{and}\quad M\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 24 \\ -6 \end{bmatrix} \quad\text{and}\quad M\begin{bmatrix} 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 21 \\ 6 \end{bmatrix},$$
find the eigenvalues of M.

144. Calculate the determinant of a 3×3 matrix whose eigenvalues are 19, 1, and -5.

145. Expand $(2 + 3a)(5 - 4a)$.

146. Re-write $(1 + t)(8 + 3t)$ in the form __ + __t if $t^2 = 10$.

$$i^2 = -1$$

147. Re-write $(1 + i)(8 + 3i)$ in the form __ + __i, knowing that $i^2 = -1$.

148. Give the determinant of a matrix whose eigenvalues are...
(a) 4, 3, and 0. (b) $1 - 4i$, $1 + 4i$, and 2. (c) 9 and $\frac{1}{4}$.

149. Simplify each of the following: i^2, i^3, i^4, i^5, i^{15}, i^{202}, $i^{1285100}$, i^{-1}.

150. Write the following in the form $a + bi$, where a and b are real numbers.

(a) $(-6 + 5i) + (2 - 4i)$

(b) $(1 + 2i)(2 + 3i)$

(c) $(-5 + 2i) - (2 - i)$

(d) $(2 - 3i)(2 + 3i)$

(e) $(1 + i)(2 - i)(3 + 2i)$

(f) $(1 - 2i)^3$

(g) $(-2i)^6$

(h) $(1 + i)^4$

151. Write $\dfrac{1 + 2i}{2 - 3i}$ in the form $a + bi$. (Hint: $\dfrac{1 + 2i}{2 - 3i} \times \dfrac{2 + 3i}{2 + 3i}$.)

> If $z = a + bi$, where a and b are real numbers, then the **real part** of z is a, and the **imaginary part** of z is b (not bi).
>
> The **magnitude** (or **modulus**) of $z = a + bi$ is the distance between $(0, 0)$ and (a, b) on an xy-plane; it is written as $|z|$. The **argument** of z is the angle between the positive x-axis and the line from $(0, 0)$ to (a, b); it is written as $\arg(z)$.

152. What is the real part of $(5 + 6i)(2i)$?

153. (a) Calculate the length of the hypotenuse of a right triangle whose legs have lengths 1 and $\sqrt{2}$.

(b) Calculate the distance between the points $(0, 0)$ and $(1, \sqrt{2})$.

(c) Calculate the magnitude of the vector $[1, \sqrt{2}]$, often written $\big|[1, \sqrt{2}]\big|$.

(d) Calculate the magnitude of the complex number $1 + \sqrt{2}i$, often written $\big|1 + \sqrt{2}i\big|$.

154. Compute $\big|2 + 7i\big|$.

155. What is the magnitude of $\sqrt{11}\cos(\pi/8) + \sqrt{11}\sin(\pi/8)i$?

156. (a) What is the real part of $1 - \sqrt{3}\,i$?

(b) What is the imaginary part of $1 - \sqrt{3}\,i$?

(c) Compute $|1 - \sqrt{3}\,i|$.

(d) Compute $\arg(1 - \sqrt{3}\,i)$.

(e) Give values for r and θ such that $1 - \sqrt{3}\,i = r\big(\cos(\theta) + i\sin(\theta)\big)$.

157. Calculate each of the following:

(a) the real part of $2i - 7$.

(b) the imaginary part of $(3 + 2i)(5i)$.

(c) the imaginary part of 4.

(d) the imaginary part of i^2

(e) the real part of i^2

(f) $\arg(-3i)$.

(g) $\arg(5 + 5i)$.

(h) $\arg(5 - 5i)$.

> **Rectangular form:** $a + b\,i$, or $a + i\,b$, or $b\,i + a$, or similar, where a and b are real numbers and usually are simplified. If a is zero, you can skip writing "0+"..., and if $b = 0$ you can skip writing ..."$+0i$".
>
> **Polar form:** $r\cos(\theta) + r\sin(\theta)\,i$, or $r\big(\cos\theta + i\sin\theta\big)$, or similar. Requires $r \geq 0$.

158. Re-write $10\cos(-\frac{\pi}{4}) + 10\sin(-\frac{\pi}{4})i$ without trig functions.

159. Write each of the following in polar form.

That is, write each as __$\big(\cos(__) + i\sin(__)\big)$, where the angles for cosine and sine must be equal and the first blank must be positive.

(a) $1-\sqrt{3}i$

(b) $-\sqrt{5}+\sqrt{15}\,i$

(c) $3+3i$

(d) $-3i$

(e) $1+\sqrt{3}\,i$

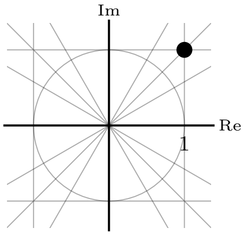
(f) $2-2\sqrt{3}\,i$

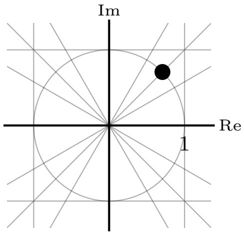
(g) $\frac{\sqrt{3}-i}{7}$

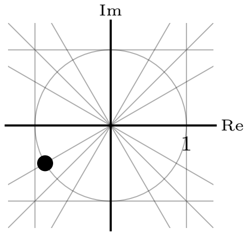
(h) $\sqrt{-1}$

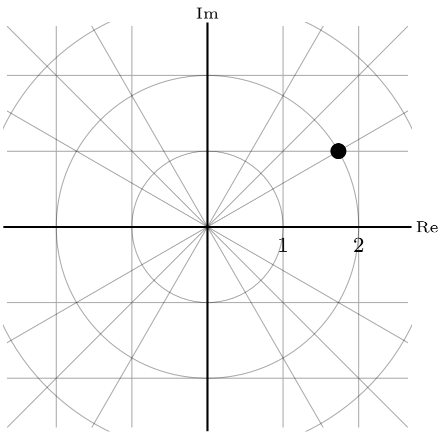
160. Write $2+\sqrt{2}\cos(3\pi/4)+\sqrt{2}\sin(3\pi/4)i$ in both rectangular and polar form.

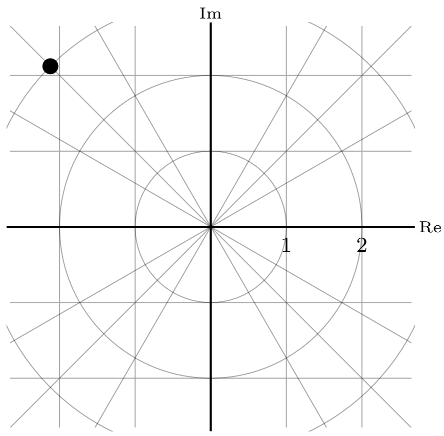
161. Write each number below in both rectangular and polar form.

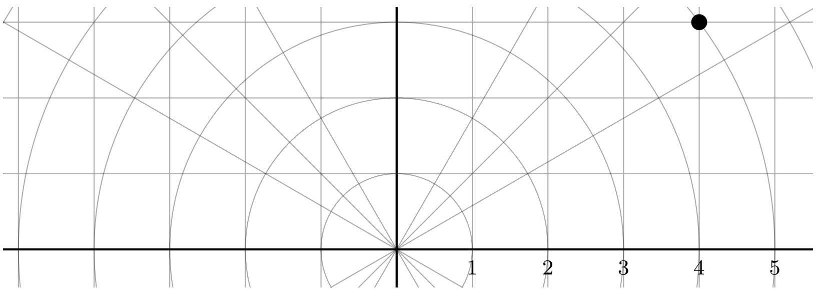
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

162. On a complex plane, *draw* the number(s)...

(a) $4-i$

(b) $\sqrt{2}-\sqrt{2}i$ and $-\sqrt{2}+\sqrt{2}i$

(c) $\dfrac{1+i}{\sqrt{2}}$ and $\dfrac{-1-i}{\sqrt{2}}$

(d) $-\dfrac{19}{29}-\dfrac{25}{29}i$

(e) $3-i$ and $3+i$

(f) $\dfrac{1-25i}{6}$

163. Re-write $(q\,n^{st})^3$ in the form __$n^{__t}$.

164. Re-write $(r\,e^{i\theta})^3$ in the form __$e^{__i}$.