Linear Algebra, Winter 2022

List 9

Review for Celebration of Knowledge 2

215. Find the eigenvalues of the matrix $M = \begin{bmatrix} -7 & 30 & 30 \\ 5 & 6 & 10 \\ -5 & 10 & 6 \end{bmatrix}$.

216. Find three non-parallel eigenvectors for the matrix from Task 215.

217. Which of the following matrices are invertible?

(a) $\begin{bmatrix} -7 & 30 & 30 \\ 5 & 6 & 10 \\ -5 & 10 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -8 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -4 \\ -5 & 10 \end{bmatrix}$

☆218. Give an example of a matrix whose eigenvalues are 2 and 7.

219. The eigenvalues of a 6×6 real matrix include

$$2, \quad -4, \quad i, \quad 2+i.$$

What is the determinant of this matrix?

220. For the complex numbers

$$z = 2\sqrt{3}e^{(\pi/3)i} = 2\sqrt{3}e^{60^\circ i} = \sqrt{3} + 3i \quad \text{and}$$
$$w = 2e^{(\pi/6)i} = 2e^{30^\circ i} = \sqrt{3} + i,$$

compute $\frac{z}{w}$, giving your answer in rectangular or exponential form (your choice).

221. For the numbers z and w from Task 220 compute $z - w$, giving your answer in rectangular or exponential form (your choice).

222. For the number z and w from Task 220 compute $|\,\overline{z} \cdot w\,|$, giving your answer in rectangular or exponential form (your choice).

223. True or false?

(a) $|z+w| = |z| + |w|$

(b) $|zw| = |z| + |w|$

(c) $|zw| = |z| \cdot |w|$

(d) $\arg(z+w) = \arg(z) + \arg(w)$

(e) $\arg(zw) = \arg(z) + \arg(w)$

(f) $\arg(zw) = \arg(z) \cdot \arg(w)$

(g) $\overline{z+w} = \overline{z} + \overline{w}$

(h) $\overline{zw} = \overline{z} + \overline{w}$

(i) $\overline{zw} = \overline{z} \cdot \overline{w}$

(j) $\sqrt{z+w} = \sqrt{z} + \sqrt{w}$

(k) $\sqrt{zw} = \sqrt{z} + \sqrt{w}$

(ℓ) $\sqrt{zw} = \sqrt{z} \cdot \sqrt{w}$

224. Convert the following numbers to exponential form, that is, $\_\_ e^{(\_\_ i)}$ where both blanks are real numbers and the first blank is non-negative.

(a) $4\cos(21^\circ) + 4\sin(21^\circ)i$

(b) $\cos(\pi/4) + \sin(\pi/4)i$

(c) $\sqrt{2}\cos(\pi/4) + \sqrt{2}\sin(\pi/4)i$

(d) $1+i$

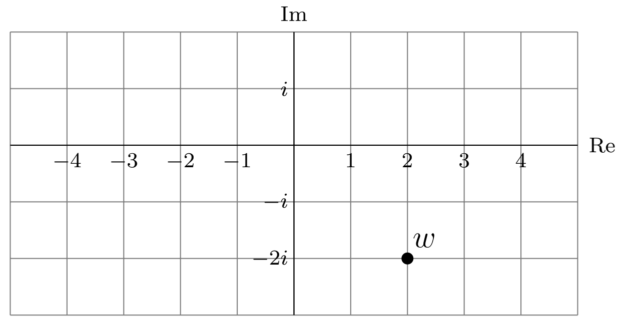
(e) $4i$

(f) $\overline{-5-5i}$ (i) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (ℓ) $\sqrt{i}$

(g) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (j) $\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^6$ (m) $\sqrt{3}-3i$

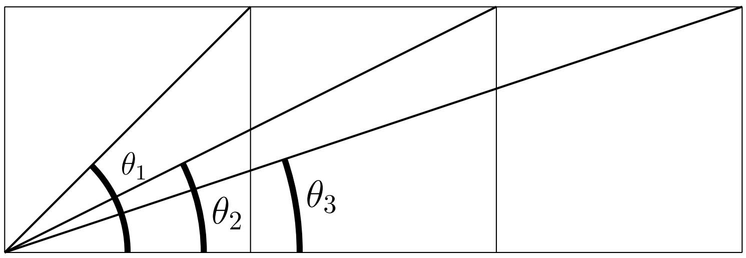
(h) $\overline{6e^{5\pi/6}}$ (k) $1-\sqrt{3}\,i$ ☆(n) $\sin(i)$

225. The figure below shows a complex number w.



Calculate $\overline{w^2}$.

☆226. The figure below shows three adjacent squares with line segments connecting some vertices. Use complex numbers to calculate the sum $\theta_1+\theta_2+\theta_3$ of the labeled angles.



☆227. The vertices of a regular octagon lie on a circle of radius 1. What is the length of each side of the octagon?

228. One of the roots of

$$z^3+2z^2-38z+80$$

is $3-i$.

(a) Find all the complex roots of this polynomial.

(b) Write $z^3+2z^2-38z+80$ as a product of irreducible complex factors.

(c) Write $x^3+2x^2-38x+80$ as a product of irreducible real factors.

229. Find the real root(s) of x^8+x^6 and the multiplicity of each root.

230. Find the complex root(s) of x^8+x^6 and the multiplicity of each root.

231. Give the cubic polynomial

$$f(x)=x^3+\_\_x^2+\_\_x+\_\_$$

for which $f(9)=0$ and for which 13 is a zero with multiplicity 2.

232. One of the roots of

$$P(x) = 2x^5 - 5x^4 + 10x^2 - 10x + 3$$

is 1. What is its multiplicity?

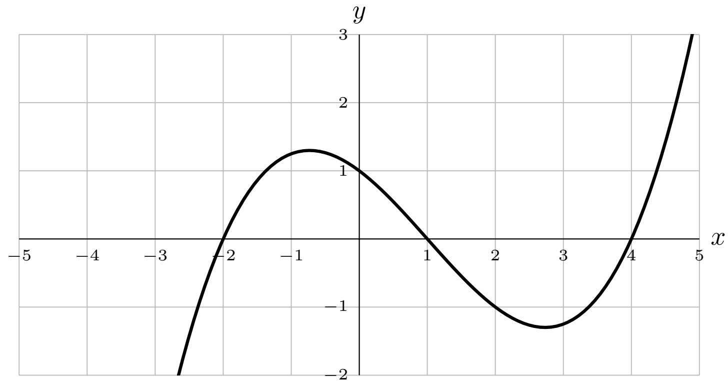
233. The polynomial

$$3z^6 - 25z^5 + 62z^4 - 132z^3 + 89z^2 - 107z + 30$$

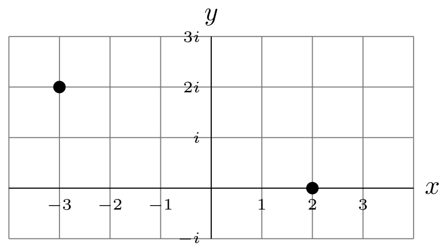
has no repeated roots. How many linear complex polynomials are factors of this polynomial?

☆234. Find all roots of $3z^6 - 25z^5 + 62z^4 - 132z^3 + 89z^2 - 107z + 30$.

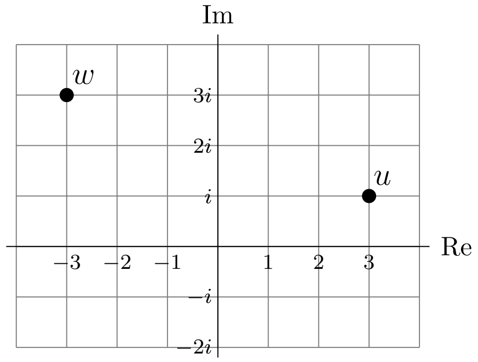
235. Give the real polynomial of the form __ $x^3 +$ __ $x^2 +$ __ $x +$ __ whose graph is shown below.



236. Give the real polynomial of the form $x^3 +$ __ $x^2 +$ __ $x +$ __ whose roots include the two complex numbers shown below.



237. Two complex numbers are shown in the figure below:



(a) Write the number u in rectangular form.

(b) Write the number w in exponential form.

(c) Calculate the complex conjugates $\overline{u}$ and $\overline{w}$.

(d) Give an example of a polynomial with real coefficients whose only zeros are u and w, or state that such a polynomial does not exist.

(e) Give an example of a polynomial with complex coefficients whose only zeros are u and w, or state that such a polynomial does not exist.

(f) Give an example of a degree-4 polynomial with real coefficients whose zeros include u and w (and possibly other points), or state that such a polynomial does not exist.