Analysis 2

3 April 2024

Task 1: $\int_0^1 15e^{5y}\mathrm{d}y = 3e^5 - 3$

Task 2: $\int_0^1 3xe^{xy}\mathrm{d}y = 3e^x - 3$

This is "new", but you should be able to think $(?)'_y = 3xe^{xy}$ and get $(3e^{xy})'_y = 3xe^{xy}$ in your head.

Task 3: $\int_0^8 (3e^x - 3)\mathrm{d}x = 3e^8 - 27$

This is 100% an Analysis 1 task.

Task 4: $\int_0^8 \int_0^1 3xe^{xy}\mathrm{d}y\mathrm{d}x = 3e^8 - 27$

This is clearly new, but it's exactly Task 2 followed by Task 3.

Iterated Integrals

An **iterated integral** requires evaluating one integral after another. They will always be definite integrals.

- The "inside" integral can give a formula as its answer.
- The "outside" integral will usually give a number as the answer.

In the example $\int_0^8 \int_0^1 3xe^{xy}\,dy\,dx$ from the previous slide,

- Inside: $\int_0^1 3xe^{xy}\,dy = 3e^{xy}\Big|_{y=0}^{y=1} = 3e^{x\cdot 1} - 3e^{x\cdot 0} = 3e^x - 3.$
- Outside: $\int_0^8 (3e^x - 3)\,dx = 3e^x - 3x\Big|_{x=0}^{x=8} = 3e^8 - 27.$

- The “inside” integral can give a formula as its answer.

- The “outside” integral will usually give a number as the answer.

Always write “x = …” or “y = …” when using a vertical line for subtraction.

- $\cos(x)\Big|_0^{\pi}$ clearly means $\cos(x)\Big|_{x=0}^{x=\pi} = \cos(\pi) - \cos(0)$, but

- $\cos(xy)\Big|_0^{\pi}$ does not make sense (are 0 and π values of x or of y?).

- The “inside” integral can give a formula as its answer.
  - The bounds ($\int_{\text{and here}}^{\text{here}}$) of inside integrals can include formulas!
- The “outside” integral will usually give a number as the answer.

Example 2: $\displaystyle\int_{-1}^{1}\int_{x^2}^{x} 4x^4y\,dy\,dx = \frac{8}{63}$ because

- Inside: $\displaystyle\int_{x^2}^{x} 4x^4y\,dy = 2x^4y^2\Big|_{y=x^2}^{y=x} = 2x^4x^2 - 2x^4(x^2)^2 = 2x^6 - 2x^8.$
- Outside: $\displaystyle\int_{-1}^{1} (2x^6 - 2x^8)dx = \frac{2x^7}{7} - \frac{2x^9}{9}\Big|_{x=-1}^{x=1} = \frac{8}{63}.$

Calculations of iterated integrals are never more complicated than Example 2, although of course the anti-derivatives can be harder.

Next week: what does an iterated integral *mean*, and what tasks require us to use iterated integrals to solve them?