Analysis 2

16 April 2024

Warm-up: Calculate $f_y'(x, y)$ for

$$f(x, y) = xy^2 - x^{10} + y + e^{\sin x} + 3.$$

Schedule

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
15 April	16 April (today)	17 April Problem Session	18 April	19 April	20 April ✗	21 April ✗
22 April ✡️ First Night of Passover	23 April 🥳 Exam 1	24 April *Lecture*	25 April	26 April	27 April ✗	28 April ✗
29 April	30 April ½ Lecture, ½ PS	1 May ✗ 🇪🇺 Labour Day	2 May ✗	3 May ✗ 🇵🇱 Constitution Day	4 May ✗	5 May ✗

Last week

Example 1: $\int_1^3 \int_2^5 yx^2 \, dx \, dy = \int_1^3 (39y) dy = 156.$

Example 2: $\int_1^3 \int_2^5 yx^2 \, dy \, dx = \int_1^3 \left(\frac{21}{2} x^2\right) dx = 91.$

Example 3: $\int_2^5 \int_1^3 yx^2 \, dy \, dx = \int_2^5 \left(4x^2\right) dx = 156.$

The "iterated integrals"

$$\int_1^3 \int_2^5 yx^2 \, dx \, dy \quad \text{and} \quad \int_2^5 \int_1^3 yx^2 \, dy \, dx$$

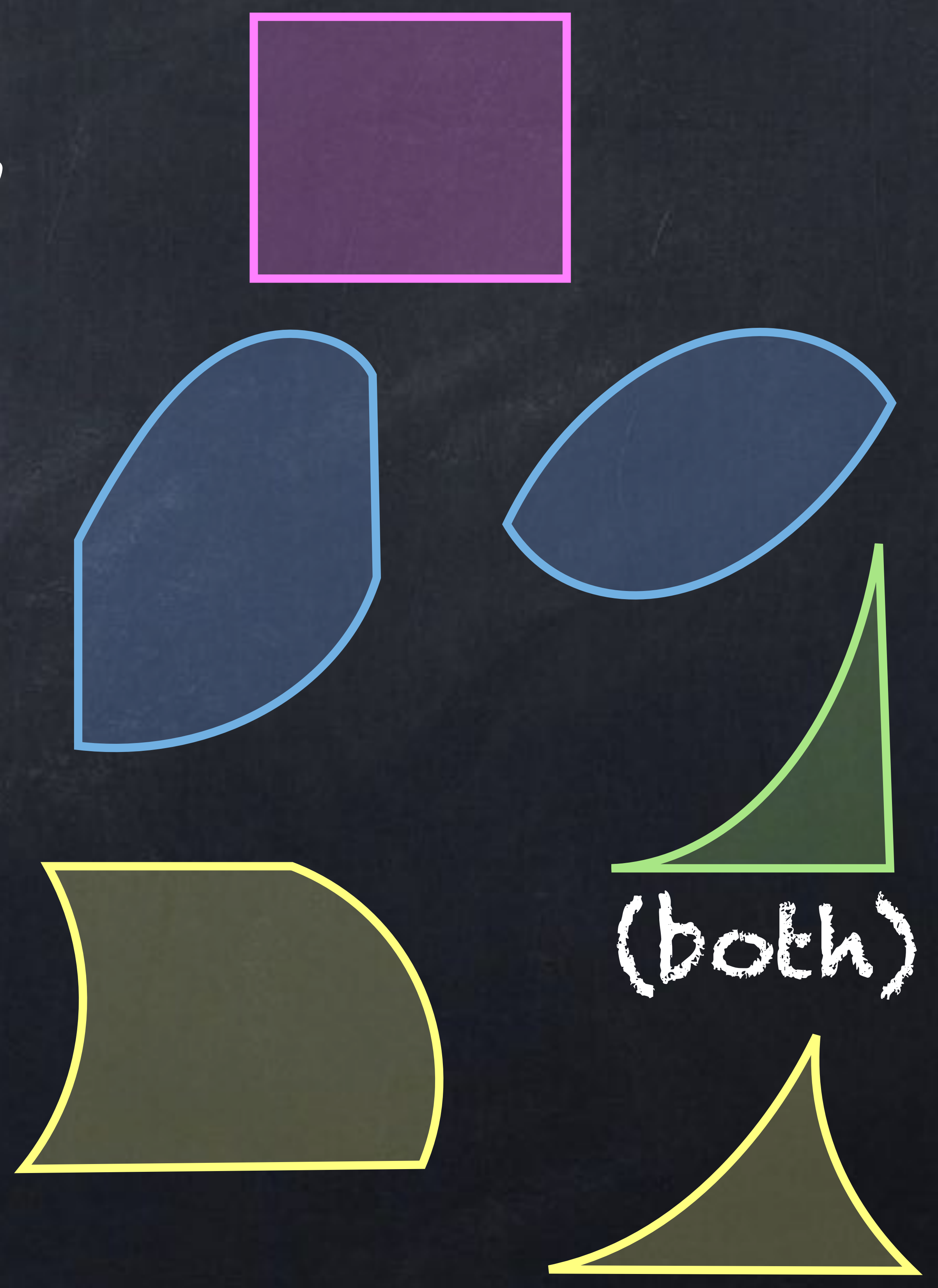
have the same value because they are both $\iint_R yx^2 dA$ for the *same rectangle* R.

Last week

Regions

The way we write an iterated integral for $\iint_D f \, dA$ depends on the shape of the region D.

- Rectangle: $\displaystyle\int_{\text{left}}^{\text{right}} \int_{\text{bot. (number)}}^{\text{top (number)}} f \, dy \, dx$ or $\displaystyle\int_{\text{bot.}}^{\text{top}} \int_{\text{left}}^{\text{right}} f \, dx \, dy$
- L/R sides are walls (or points): $\displaystyle\int_{\text{left}}^{\text{right}} \int_{\text{bottom fn.}}^{\text{top function}} f \, dy \, dx$
- Top and bottom are flat (or points): $\displaystyle\int_{\text{bot.}}^{\text{top}} \int_{\text{left fn.}}^{\text{right fn.}} f \, dx \, dy$



Anti-derivatives

In Analysis 1, both definite integrals $\int_0^1 x^2 \mathrm{d}x$ and indefinite integrals $\int x^2 \mathrm{d}x$ are very common.

- $F(b) - F(a)$ can be a meaningful physical quantity.
- $F(x) + C$ isn't actually that useful but itself, but doing indefinite integrals is good practice for definite integrals.

It's uncommon to see $\int f(x, y)\mathrm{d}x$ or $\int f(x, y)\mathrm{d}y$ as an indefinite integral task.

This is because it doesn't answer a useful task by itself.

Anti-derivatives

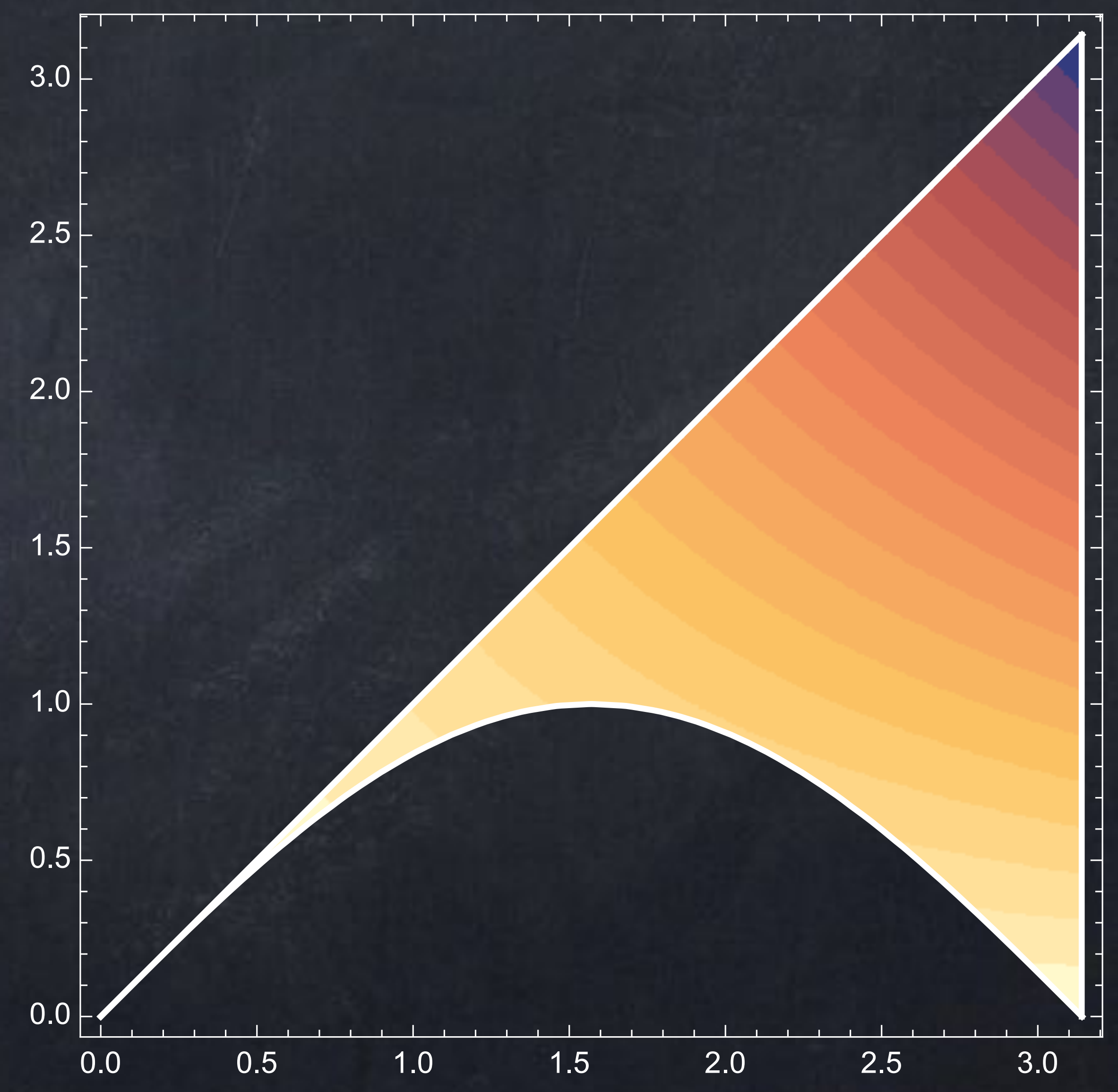
Does the answer to this task have a helpful science/engineering application?

✅ Calculate $\displaystyle\int_0^{\pi}\int_{\sin x}^{x}(2xy+1)\mathrm{d}y\mathrm{d}x.$

✅ Calculate $\displaystyle\int_{\sin x}^{x}(2xy+1)\mathrm{d}y.$

❎ Calculate $\displaystyle\int(2xy+1)\mathrm{d}y.$

mass of



(color is density)

Anti-derivatives

Does the answer to this task have a helpful science/engineering application?

✅ Calculate $\int_0^\pi \int_{\sin x}^{x} (2xy + 1)\mathrm{d}y\mathrm{d}x$.

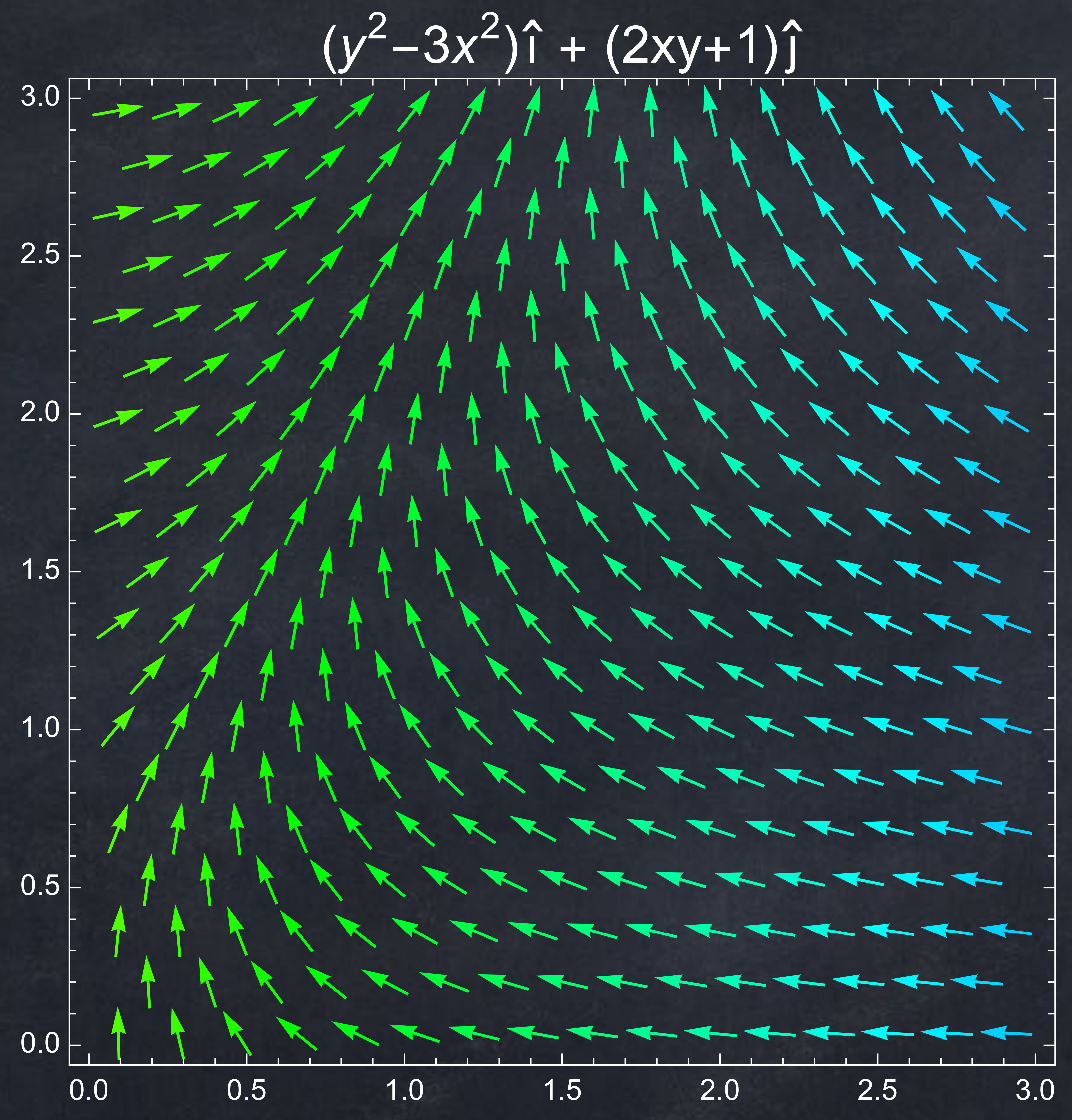
☑️ Calculate $\int_{\sin x}^{x} (2xy + 1)\mathrm{d}y$.

❌ Calculate $\int (2xy + 1)\mathrm{d}y$.

❌ Describe all functions $f(x, y)$ for which $f_y' = 2xy + 1$.

These are the same task.

✅ Describe all functions $f(x, y)$ for which $\nabla f = \begin{bmatrix} y^2 - 3x^2 \\ 2xy + 1 \end{bmatrix}$.

If $\vec{E} =$  then voltage $= -f$.

✅ Describe all functions $f(x, y)$ for which $\nabla f = \begin{bmatrix} y^2 - 3x^2 \\ 2xy + 1 \end{bmatrix}$.

Task: Describe all functions $f(x, y)$ for which $\nabla f = \begin{bmatrix} y^2 - 3x^2 \\ 2xy + 1 \end{bmatrix}$.

f(x,y) will have the format

$$\int (2xy+1)\,dy = xy^2 + y + g(x)$$

for some yet-unknown g(x),
(not just $xy^2 + y + C$).

Warm-up: Calculate $f_y'(x, y)$ for

$$f(x, y) = xy^2 - x^{10} + y + e^{\sin x} + 3.$$

$$f'_y = 2xy + 1$$

Task: Describe all functions $f(x, y)$ for which $\nabla f = \begin{bmatrix} y^2 - 3x^2 \\ 2xy + 1 \end{bmatrix}$.

$f(x,y)$ will have the format

$$\int (2xy+1)\,dy = xy^2 + y + g(x)$$

for some yet-unknown $g(x)$.

From the formula above, $f'_x = y^2 + 0 + g'(x)$.

But from ∇f we also know $f'_x = y^2 - 3x^2$.

So it must be that $g'(x) = -3x^2$. Thus $g(x) = -x^3 + C$, and

$$\boxed{f(x,y) = xy^2 + y - x^3 + C.}$$

A: Integrate $f(x, y) = xy\cos(x^2)$ over the 3×12 (width × height) rectangle with $(0, 1)$ as its lower-left corner.

Answer: $42\sin(9)$

B: Integrate $f(x, y) = y$ over the region bounded by $x = 0$, $x = 1$, $y = x^2$, and $y = x^2 + 1$.

Answer: 5/6

C: Integrate $f(x, y) = 1$ over the region bounded by $y = x + 5$, $y = \pi$, $y = -\pi$, and $x = \sin(y)$.

Answer: 10π

D: Integrate $f(x, y) = 1$ over the region bounded by $y = 5 - x^2$, $y = (x+1)^2$.

Answer: 9

E: Integrate $f(x, y) = 5x$ over the right half-disk of radius 1 centered at the origin.

Answer: 10/3

F1: Integrate $f(x, y) = x^3y^2$ over the region bounded by the x-axis, the line $x = 1$, and the curve $y = x^2$.

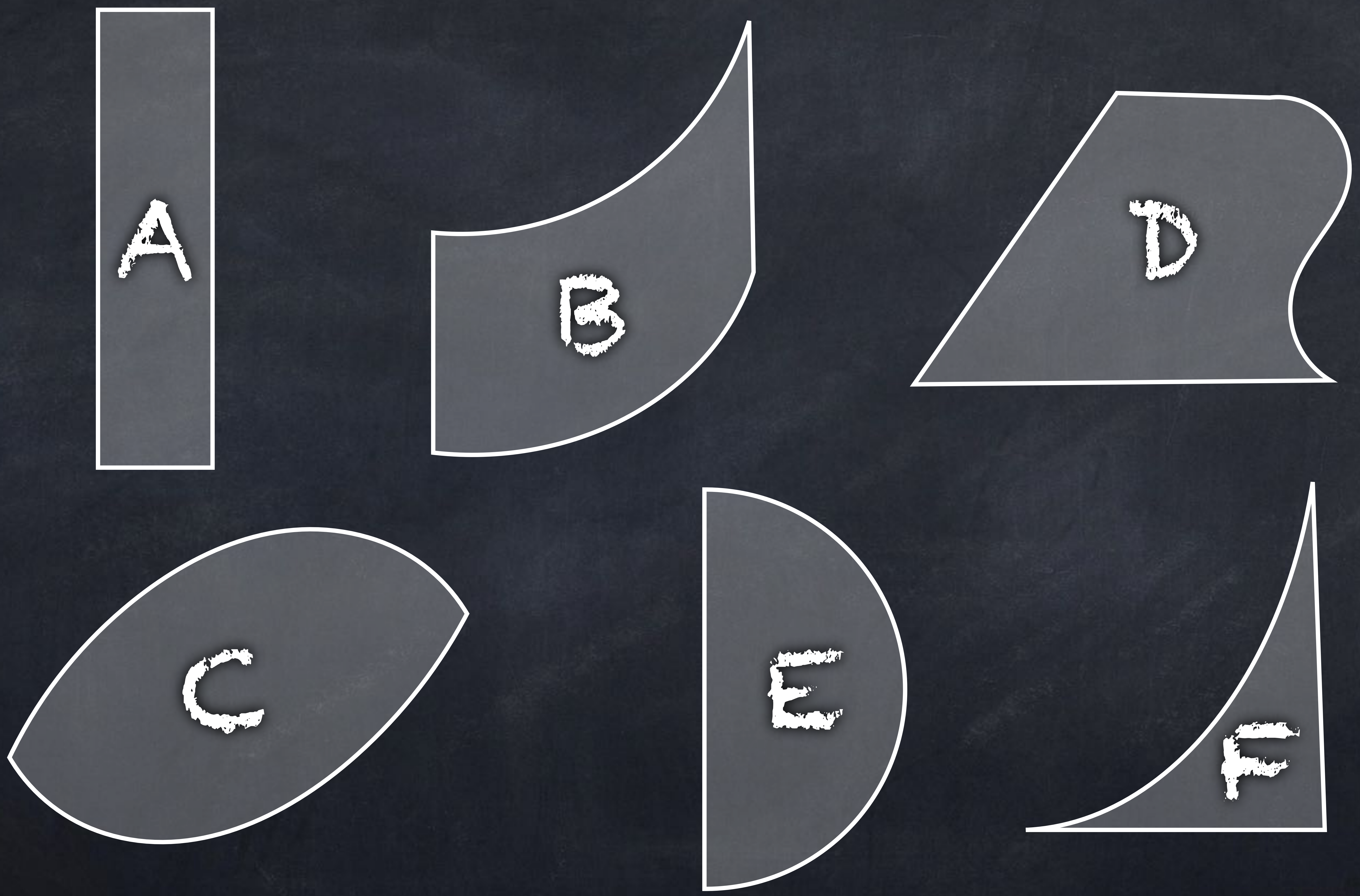
Answer: 1/30

F2: Integrate $f(x, y) = e^{(x^3)}$ over the region bounded by the x-axis, the line $x = 1$, and the curve $y = x^2$.

Answer: (e-1)/3

F3: Integrate $f(x, y) = (1 + \sqrt{y}) \cdot \sin(y)$ over the region bounded by the x-axis, the line $x = 1$, and the curve $y = x^2$.

Answer: 1-sin(1)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