List 0

"Previous topics"

—————————————— Basic algebra ——————————————

1. Find the two values of r for which $r^2 - 2r - 15 = 0$.

2. Find the two (complex) values of r for which $r^2 - 2r + 15 = 0$.

3. Solve $\ln(y) = \sin(x)$ for y.

4. Solve $\ln(x) - 3 = 7t$ for x.

5. Solve $\dfrac{-1}{2y^2} = C + \sqrt{x^2 + 1}$ for y.

6. Solve $e^y = 9\sin(3t) - t^2 + C$ for y.

7. Find the value of C for which $\dfrac{1}{2} = \dfrac{-3}{1 + C}$.

8. Find the real value of C for which $\sin(0) = 10Ce^0 - \dfrac{2}{25C^2}$.

9. Find values of A and B such that
$$A \cdot (x + 6) + B \cdot (x - 2) = 2x - 6.$$

10. Find values of C_1 and C_2 such that both of these equations are true:
$$2C_1 + 2C_2e^0 - 3\sin(0) - 0\sin(0) + 0\cos(t) = 0,$$
$$2C_2e^0 - 3\cos(0) + -0\sin(0) - \sin(0) - 0\cos(0) + \cos(0) = 1.$$

11. If $y(x) = \dfrac{-1}{\sqrt{C - 2\sqrt{x^2 + 1}}}$ and $y(0) = -1$, find the value of C.

—————————————— Linear algebra ——————————————

12. Calculate the length (also called magnitude or norm) of the vector $5\hat{\imath} + \hat{\jmath} + 5\hat{k}$.

13. Calculate $\big| [2, -3] \big|$.

14. Give a unit vector (that is, a vector of magnitude 1) that points in the same direction as the vector $\vec{v} = [15, 8] = \begin{bmatrix} 15 \\ 8 \end{bmatrix} = 15\hat{\imath} + 8\hat{\jmath}$.

15. Give a unit vector that points in the same direction as $5\hat{\imath} - 2\hat{\jmath}$.

16. Calculate the dot product (also called scalar product) of the vectors $\vec{u} = [0, 1]$ and $\vec{v} = [-8, 5]$.

17. If $|\vec{v}| = 8$ and $|\vec{w}| = 7$ and the angle between $\vec{v}$ and $\vec{w}$ is $120° = \frac{2}{3}\pi$, what is the value of $\vec{v} \cdot \vec{w}$?

18. If $|\vec{v}| = 3$ and $|\vec{n}| = 16$ ...

    (a) ... and $\vec{v}$ points in the same direction as $\vec{n}$, what is the value of $\vec{v} \cdot \vec{n}$?

    (b) ... and $\vec{v}$ is perpendicular to $\vec{n}$, what is the value of $\vec{v} \cdot \vec{n}$?

    (c) ... and $\vec{v}$ points in the exact opposite direction as $\vec{n}$ (this is sometimes called "anti-parallel"), what is the value of $\vec{v} \cdot \vec{n}$?

19. If $|\vec{u}| = 1$ and $|\vec{v}| = 4$,

    (a) is it possible that $\vec{u} \cdot \vec{v} = 2\sqrt{3}$?

    (b) is it possible that $\vec{u} \cdot \vec{v} = 2$?

    (c) is it possible that $\vec{u} \cdot \vec{v} = -2$?

    (d) is it possible that $\vec{u} \cdot \vec{v} = 3.81$?

    (e) is it possible that $\vec{u} \cdot \vec{v} = 4.61$?

    (f) is it possible that $\vec{u} \cdot \vec{v} = -\sqrt{17}$?

    (g) is it possible that $\vec{u} \cdot \vec{v} = -\sqrt{7}$?

20. If $|\vec{u}| = 1$ and $|\vec{w}| = 7$, describe ALL possible values that $\vec{u} \cdot \vec{w}$ could have.

21. If $|\vec{u}| = 1$ and $\vec{n} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$,

    (a) what is the largest possible value that $\vec{u} \cdot \vec{n}$ could have?

    (b) give an example of a vector $\vec{u}$ such that $\vec{u} \cdot \vec{n}$ has the value from part (a).

    (c) give an example of a vector $\vec{u}$ such that $\vec{u} \cdot \vec{n} = 0$.

22. Write $\dfrac{5x + 6}{x^2 - 6x + 8} = \dfrac{5x + 6}{(x - 2)(x - 4)}$ as a sum of partial fractions.
That is, find A and B such that
$$\frac{5x + 6}{x^2 - 6x + 8} = \frac{A}{x - 2} + \frac{B}{x - 4}.$$

23. Write $\dfrac{2x - 6}{(x - 2)(x + 6)}$ as a sum of partial fractions.

24. Write $\dfrac{36}{x^3 + 9x^2 + 18x}$ as a sum of partial fractions.

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25. Give the derivative (with respect to t) of $y = 2e^{3t} + 4\sin(5t) + 6\cos(7t) + 8t^9 + 10$.

26. If $y = x^9$, calculate $\dfrac{dy}{dx} + y'(x) + y'$.

27. If $y = 5\,e^x \sin(\sqrt{14}x)$, simplify $y'' - 2y' + 15y$ as much as possible.

28. Find all critical points of $f(x) = x^4 - 4x^3 - 8x^2 + 2$ and classify each one as a local minimum, local maximum, or neither.

29. Find and classify the critical points of $f(x) = e^{x^2}(2x + 3)$.

30. If $f(3) = 5$, $f'(3) = 0$, and $f''(3) = 2$, could $x = 3$ be a local minimum of $f(x)$? Could it be a local maximum?

31. Find the following indefinite integrals.

(a) $\int 11\, x^4\, \mathrm{d}x$

(b) $\int x^{-1/2}\, \mathrm{d}x$

(c) $\int (\sin(2x))^2 \cos(2x)\, \mathrm{d}x$

(d) $\int \dfrac{x^4}{\sqrt{x^5 + 1}}\, \mathrm{d}x$

(e) $\int 11\, y^4\, \mathrm{d}y$

(f) $\int \dfrac{1}{y^3}\, \mathrm{d}y$

(g) $\int \dfrac{1}{y^2}\, \mathrm{d}y$

(h) $\int \dfrac{1}{y}\, \mathrm{d}y$

(i) $\int e^{6t}\, \mathrm{d}t$

(j) $\int t e^t\, \mathrm{d}t$

(k) $\int e^{-x}(2x - 3)\, \mathrm{d}x$

32. Find the definite integral $\int_0^1 (4x^3 - 9x^2)\, \mathrm{d}x$. (Your answer should be a number.)

33. Give the definite integral $\int_0^1 (4x^3 - 9x^2 k^2)\, \mathrm{d}x$. (Your answer should be a formula with k.)

34. Calculate $\int_a^b x\, \mathrm{d}x$. (Your answer should be a formula with a and b.)

35. Calculate $\int_{q^2}^{\sin q} x\, \mathrm{d}x$. (Your answer should be a formula with q.)

36. Calculate (a) $\int_0^3 x e^{2x}\, \mathrm{d}x$, (b) $\int_0^3 t e^{2t}\, \mathrm{d}t$, (c) $\int_0^3 y e^{2y}\, \mathrm{d}y$.