Analysis 2, Summer 2024

List 10

Systems of linear ODEs / Review for Exam 2

A **system of two first-order linear ODEs** for $x(t)$ and $y(t)$ can be written as
$$\begin{cases} x' = a_{11}x + a_{12}y + b_1 \\ y' = a_{21}x + a_{22}y + b_2 \end{cases}$$
In this course, the systems will always be homogeneous (meaning $b_1 = b_2 = 0$) and the coefficients a_{ij} will always be constants.

241. (a) Re-write the system
$$\begin{cases} x' = 3y \\ y' = x + 2y \end{cases}$$
as a single second-order linear ODE for $x(t)$. Simplify to remove fractions.

Re-write the first equation as $y = \frac{1}{3}x'$, then since $y' = \frac{1}{3}x''$ the second equation becomes $(\frac{1}{3}x'') = x + 2(\frac{1}{3}x')$, or
$$\tfrac{1}{3}x'' - \tfrac{2}{3}x' - x = 0 \qquad \text{or} \qquad \boxed{x'' - 2x' - 3x = 0}.$$

(b) Re-write the system as a single ODE for $y(t)$.

From the second equation, $x = y' - 2y$, so $x' = y'' - 2y'$ and then the first equation becomes $y'' - 2y' = 3y$, or $\boxed{y'' - 2y' - 3y = 0}$.

(c) Find the general solution to the system.

Elimination of y: The ODE $x'' - 2x' - 3x = 0$ has characteristic polynomial
$$f(\lambda) = \tfrac{1}{3}\lambda^2 - \tfrac{2}{3}\lambda - 1 = \tfrac{1}{3}(\lambda + 1)(\lambda - 3),$$
which has roots -1 and 3, so $x = C_1e^{-t} + C_2e^{3t}$. Recall that $y = \frac{1}{3}x'$, so $y = \frac{1}{3}(-C_1e^{-t} + 3C_2e^{3t}) = \frac{-1}{3}C_1e^{-t} + C_2e^{3t}$. The general solution to the system is therefore $\boxed{x = C_1e^{-t} + C_2e^{3t}, y = \frac{-1}{3}C_1e^{-t} + C_2e^{3t}}$.

Elimination of x: Using $x = y' - 2y$, we get $y'' - 2y' - 3y' = 0$ and thus $y = C_1e^{-t} + C_2e^{3t}$. Then $x = y' - 2y = (C_1e^{-t} + C_2e^{3t})' - 2(C_1e^{-t} + C_2e^{3t}) = -3C_1e^{-t} + C_2e^{3t}$. This gives $\boxed{x = -3C_1e^{-t} + C_2e^{3t}, y = C_1e^{-t} + C_2e^{3t}}$. It may look different, but this *is* equivalent to the first answer when "C_1" is replaced by $-3C_1$.

Eigenvectors: (This is another method you can use if you want.) The general solution is always
$$\begin{bmatrix} x \\ y \end{bmatrix} = \vec{v}_1e^{\lambda_1 t} + \vec{v}_2e^{\lambda_2 t},$$
where $\lambda_i, \vec{v}_i$ are eigenvalue-eigenvector pairs of the coefficient matrix. For this task, $A = \left[\begin{smallmatrix} 0 & 3 \\ 1 & 2 \end{smallmatrix}\right]$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$ with corresponding eigenvectors $\vec{v}_1 = C_1\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $\vec{v}_2 = C_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (remember that eigenvectors are really any scalar multiple of that vector; that's why we can write C_1 and C_2 as part of the vectors). The solution to ODE is
$$\begin{bmatrix} x \\ y \end{bmatrix} = C_1\begin{bmatrix} -3 \\ 1 \end{bmatrix}e^{-t} + C_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{3t} = \boxed{\begin{bmatrix} -3C_1e^{-t} + C_2e^{3t} \\ C_1e^{-t} + C_2e^{3t} \end{bmatrix}}.$$

242. Solve $\begin{cases} x' = 12x + 2y \\ y' = -12x + 23y. \end{cases}$ $\boxed{\begin{cases} x = C_1 e^{20t} + 2C_2 e^{15t} \\ y = 4C_1 e^{20t} + 3C_2 e^{15t} \end{cases}}$

The version above is written so that none of the coefficients use fractions. There are many other (equivalent) correct answers. For example, replacing C_2 with $\frac{1}{2}C_2$, we get

$$\begin{cases} x = C_1 e^{20t} + C_2 e^{15t} \\ y = 4C_1 e^{20t} + \frac{3}{2}C_2 e^{15t}, \end{cases}$$

which would be the result of eliminating y. If you eliminate x instead, then

$$\begin{cases} x = \frac{1}{4}C_1 e^{20t} + \frac{2}{3}C_2 e^{15t} \\ y = C_1 e^{20t} + C_2 e^{15t} \end{cases}$$

might be the format you get.

243. Solve $\begin{cases} x' = 3x - 2y \\ y' = 4x + 7y, \end{cases}$ $\quad x(0) = 8, \quad y(0) = -8.$

General solution $\begin{cases} x = C_1 e^{5t}\sin(2t) + C_2 e^{5t}\cos(2t) \\ y = -C_1 e^{5t}\sin(2t) + C_2 e^{5t}\sin(2t) - C_1 e^{5t}\cos(2t) - C_2 e^{5t}\cos(2t) \end{cases}$,

so $\begin{cases} x(0) = C_2 \\ y(0) = -C_1 - C_2. \end{cases}$ Thus $C_1 = 0$, $C_2 = 8$, and $\boxed{\begin{cases} x = 8e^{5t}\cos(2t) \\ y = 8e^{5t}\sin(2t) - 8e^{5t}\cos(2t) \end{cases}}$

☆244. Solve $\begin{cases} x' = x - 2y - z \\ y' = -x + y + z \\ z' = x - z. \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C_1 \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} e^{-t} + C_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} e^{0t} = \boxed{\begin{bmatrix} 3C_1 e^{2t} + C_3 \\ -2C_1 e^{2t} - C_2 e^{-t} \\ C_1 e^{2t} + 2C_2 e^{-t} + C_3 \end{bmatrix}}$$

245. Solve $\begin{cases} x' = -2x + y \\ y' = 3x - 2y \end{cases}$

$r^2 + 4r + 1$ has roots $-2 \pm \sqrt{3}$. $\boxed{\begin{cases} x = C_1 \frac{1}{\sqrt{3}} e^{(-2+\sqrt{3})t} + C_2 \frac{-1}{\sqrt{3}} e^{(-2-\sqrt{3})t} \\ y = C_1 e^{(-2+\sqrt{3})t} + C_2 e^{(-2-\sqrt{3})t}. \end{cases}}$

246. Complete the following table:

	Separable?	Homogeneous linear?	Non-homogeneous linear?
$y' = \sin(y)$	Yes	No	No
$y' = \sin(x)$	Yes	No	Yes
$x' = \sin(t)$	Yes	No	Yes
$x' = \sin(x)$	Yes	No	No
$y' = y \cdot t$	Yes	Yes	No
$y' = y + t$	No	No	Yes
$y' = y$	Yes	Yes	No
$y' = t$	Yes	No	Yes

247. Is the ordinary differential equation $y' = x^3$

(a) first-order? $\boxed{\text{yes}}$ (b) second-order? $\boxed{\text{no}}$ (c) third-order? $\boxed{\text{no}}$

(d) direct? yes

(e) autonomous? no

(f) separable? yes

(g) linear? yes

(h) linear and homogeneous? no

(i) linear with constant coefficients? yes

248. Solve the direct ODE $f' = 3x\cos(3x) + \sin(3x)$ for $f(x) =$ $x\sin(3x) + C$

249. Solve the direct PDE $f'_x = \sin(xy) + xy\cos(xy)$, $f'_y = x^2\cos(xy) + 2y$ for $f(x, y) =$ $x\sin(xy) + y^2 + C$

250. Does the partial differential equation $f'_x = x^2y, f'_y = x^3y$ have a solution? Why or why not?

No, it does not because $f''_{xy} = f''_{yx}$ for any smooth function $f(x, y)$, and yet $f''_{xy} = (f'_x)'_y = (x^2y)'_y = x^2$ and $f''_{yx} = (f'_y)'_x = (x^3y)'_x = 3x^2y$ according to the equation in the task. Since x^2 and $3x^2y$ are not exactly equal, there can be no smooth function f for which $f'_x = x^2y$ and $f'_y = x^3y$.

251. Solve the following PDEs, if solutions exist:

(a) $f'_x = x, \quad f'_y = y$ $f = \frac{1}{2}x^2 + \frac{1}{2}y^2 + C$

(b) $f'_x = x, \quad f'_y = x$ No solution exists because f would need to satisfy $f''_{xy} = (f'_x)'_y = (x)'_y = 0$ and $f''_{yx} = (f'_y)'_x = (x)'_x = 1$.

(c) $f'_x = y, \quad f'_y = x$ $f = xy + C$

(d) $f'_x = x^2y^2, \quad f'_y = \frac{2}{3}x^3y$ $f = \frac{1}{3}x^3y^2 + C$

(e) $f'_x = 3x^2 + y, \quad f'_y = 6x + 5y^4$ No solution exists because $1 \neq 6$.

(f) $f'_x = 3x^2 + y, \quad f'_y = x + 5y^4$ $f = x^3 + xy + y^5 + C$

(g) $f'_x = 6xy\,e^{x^2y}, \quad f'_y = 3x^2e^{x^2y}$ $f = 3e^{x^2y} + C$

(h) $f'_x = \dfrac{\sin(y)}{x}, \quad f'_y = \cos(y)\ln(x)$ $f = \ln(x)\sin(y) + C$

(i) $f'_x = x^2 + \dfrac{1}{y}, \quad f'_y = x^3 - \dfrac{x}{y^2}$ No solution exists because $\frac{-1}{y^2} \neq 3x^2 - \frac{1}{y^2}$.

252. The general solution to the ODE

$$y' + \frac{y}{x} = x^2y^2$$

is $y = \dfrac{2}{Cx - x^3}$. Knowing this, give the particular solution to the IVP

$$y' + \frac{y}{x} = x^2y^2, \qquad y(1) = 3.$$

$\dfrac{2}{C-1} = 3$ means $C = \frac{5}{3}$, so $y = \dfrac{2}{\frac{5}{3}x - x^3}$ or, simplified, $y = \dfrac{6}{5x - 3x^3}$

253. Is $y = x^3y^3$ a general solution or particular solution? $\boxed{\text{particular}}$ Is it an implicit solution or an explicit solution? $\boxed{\text{implicit}}$ (The explicit would be $y = \pm\sqrt{x^{-3}}$.)

254. Give the order of the differential equation

$$x^{11}y'' + e^x y = \sin(e^x).$$

(Do *not* try to solve this ODE.) $\boxed{\text{order 2}}$

255. Which of the following ODEs is third-order? $\boxed{\text{B}}$
(A) $x' = x^3$ (B) $x''' = \sin(x)$ (C) $y' + y'' = 3$ (D) $y'' = t^3 - 4t^2 + 1$

256. The general solution to the ODE

$$x' + x = \frac{2t}{x}$$

is $x = \pm\sqrt{Ce^{-2t} + 2t - 1}$. Knowing this, give the particular solution to the IVP

$$x' + x = \frac{2t}{x}, \qquad x(0) = -4.$$

$\boxed{x = -\sqrt{17e^{-2t} + 2t - 1}}$

257. Give an example of a first-order ODE for $y = y(t)$ in each of the following categories: These are just my examples. Yours might look very different.

(a) direct, $y' = t^3$

(b) autonomous, $y' = \sin(y)$

(c) separable but not direct or autonomous, $y' = e^t\cos(y)$

(d) homogenous linear with constant coefficients, $2y' - 7y = 0$

(e) homogenous linear with non-constant coefficients, $y' + t^2y = 0$

(f) non-homogenous linear with constant coefficients, $2y' - 7y = 1$ or $y' + 3y = t^2$

(g) non-homogenous linear with non-constant coefficients. $y' + \ln(t)y = 5t^2$

258. Which category/categories from Task 257 does $x^2y' + y - 5 = 0$ belong to? $\boxed{\text{(c) separable}}$ because $y' = \frac{1}{x^2}(5 - y)$

and $\boxed{\text{(g) non-homogenous linear with non-constant coefficients}}$ because we can re-write it as $x^2y' + y = 5$ or as $y' + x^{-2}y = 5x^{-2}$.

259. Give an example of a homogenous second-order linear ordinary differential equation with *non*-constant coefficients.

There are many examples. A simple one is $y'' + t\,y = 0$.

260. Re-write each of the following linear ODEs into the form $y' + a(x)y = f(x)$ or $y' + a(t)y = f(t)$ or $x' + a(t)x = f(t)$.

(a) $x^2y' = 7 - y$ $\boxed{y' + x^{-2}y = 7x^{-2}}$

(b) $x' = 12x$ $\boxed{x' - 12x = 0}$

(c) $y' = 9xy$ $\boxed{y' - 9xy = 0}$

(d) $y' + t = -y$ $\boxed{y' + y = -t}$

261. Solve the following first-order ODEs:

(a) $y' = e^{(x^5)}x^4$ (Separable / direct) $\boxed{y = \frac{1}{5}e^{(x^5)} + C}$

(b) $y' = y^2$ (Separable / autonomous) $\boxed{y = \dfrac{-1}{t+C}}$ or $\boxed{y = \dfrac{-1}{x+C}}$ since the variable is not stated

(c) $y' = x^3y^2$ (Separable) $\boxed{y = \dfrac{-4}{x^4+C}}$

(d) $x' - e^t x = 5e^t$ (Linear and separable) $\boxed{x = -5 + Ce^{e^t}}$

(e) $tx' + 3x = t^3$ (Linear) $\boxed{x = \dfrac{t^3}{6} + \dfrac{C}{t^3}}$

(f) $y' = \sin(x) \cdot e^y$ (Separable) $\boxed{y = -\ln(C + \cos(x))}$

(g) $x' = \sin(t) \cdot e^x$ (Separable) $\boxed{x = -\ln(C + \cos(t))}$ is basically same as part (f)

(h) $y' = \sin(x) \cdot e^x$ (Separable / direct) $\boxed{y = \frac{1}{2}e^x(\sin(x) - \cos(x)) + C}$

(i) $y' + 3t^2y = 10t^2$ (Linear) $\boxed{y = \dfrac{10}{3} + C_1e^{-t^3}}$

(j) $y' + 2y = \sin(t)$ (Linear) $\boxed{y = \dfrac{2}{5}\sin(t) - \dfrac{1}{5}\cos(t) + Ce^{-2t}}$

(k) $y' - 5y^2 = xy^2$ (Separable) $\boxed{y = \dfrac{2}{C + 10x - x^2}}$

(ℓ) $x \cdot x' = t^3$ (Separable) $\boxed{x = \pm\sqrt{C + \frac{1}{2}t^4}}$

262. Give two different particular solutions to $y' = xy$. The general soln. is $y = Ce^{x^2/2}$, so pick any two values for C. The simplest examples are $\boxed{y = 0}$ and $\boxed{y = e^{x^2/2}}$.

263. Give the Laplace transforms of

$$f(t) + g(t) \qquad \text{and} \qquad f(t)g(t)$$

for $f(t) = 3e^{2t}$ and $g(t) = \sin(5t)$. $F(s) + G(s) = \boxed{\dfrac{3}{s-2} + \dfrac{5}{s^2+25}}$. There is no universal formula for $\mathscr{L}[fg]$, but using the general rule $\mathscr{L}[e^{2t}g(t)] = G(s-2)$ we get that $\mathscr{L}[3e^{2t}\sin(5t)] = \boxed{\dfrac{15}{(s-2)^2+25}}$.

264. Give the inverse Laplace transform of

$$F(s) = \frac{8-5s}{s^2+4}.$$

That is, what function $f(t)$ has $F(s) = \mathscr{L}[f(t)]$? $F(s) = \dfrac{8}{s^2+4} - \dfrac{5s}{s^2+4} = 2\left(\dfrac{4}{s^2+4}\right) - 5\left(\dfrac{s}{s^2+4}\right)$, so $f(t) = \boxed{4\sin(2t) - 5\cos(2t)}$.

265. Give the Laplace transform of

$$y' + 6y + 5t$$

if $y(0) = 2$. Your answer should include s and also $Y = \mathscr{L}[y]$. $\boxed{sY - 2 + 6Y + \frac{5}{s^2}}$

266. For the IVP $y' = y^3, \quad y(0) = 5$, give

(a) an implicit general solution, $\boxed{\frac{-1}{2y^2} = t + C}$

(b) an implicit particular solution, $\boxed{\frac{-1}{2y^2} = t - \frac{1}{50}}$

(c) an explicit general solution, $\boxed{y = \frac{1}{\sqrt{C - 2t}}}$

(d) the explicit particular solution. $\boxed{y = \frac{1}{\sqrt{\frac{-1}{50} - 2t}}}$ or $\boxed{y = \frac{5}{\sqrt{1 - 50t}}}$ or other ways to write the same function

267. For the ODE $y' = 2xy$, give

(a) an implicit general solution, $\boxed{\ln(y) = x^2 + C}$

(b) an implicit particular solution, $\ln(y) = x^2$ or $\ln(y) = x^2 + 123$ or others

(c) the explicit general solution, $\boxed{y = Ce^{x^2}}$

(d) an explicit particular solution. $y = e^{x^2}$ or $y = -12e^{x^2}$ or others

268. Solve the ODE $y' + 6ty = e^{-3t^2}$. $\boxed{y = e^{-3t^2}(t + C)}$

269. Solve the IVP

$$y' + \cos(x)y = 12\cos(x), \qquad y(0) = 5.$$

$\boxed{y = 12 - 7e^{-\sin(x)}}$

270. Give the particular solution to $y' - 7y = e^{8t}$ with each of the following initial conditions: (a) $y(0) = 0$. $\boxed{y = e^{8t} - e^{7t}}$ (b) $y(0) = 4$. $\boxed{y = e^{8t} + 3e^{7t}}$ (c) $y(0) = 1$. $\boxed{y = e^{8t}}$ (d) $y(1) = 0$ $\boxed{y = e^{8t} - e^{7t+1}}$

271. One of the following ODEs is autonomous. Solve that one and ignore the other.

$$x' + tx = 12 \qquad \text{or} \qquad y' + 3y = 12.$$

The y ODE is autonomous. It's not clear whether y' means $\frac{dy}{dx}$ or $\frac{dy}{dt}$, so both $\boxed{y = 4 + Ce^{-3t}}$ and $\boxed{y = 4 + Ce^{-3x}}$ are good answers.

272. One of the following ODEs is separable. Solve that one and ignore the other.

$$x' + t^3 = 5t \qquad \text{or} \qquad y' + ty = 5.$$

$\boxed{x = \frac{5t^2}{2} - \frac{t^4}{4} + C}$

273. One of the following ODEs is linear. Solve that one and ignore the other.

$$x' + 5t^4 x = t^4 \qquad \text{or} \qquad y' + 5t^4 y = y^4.$$

$$\boxed{x = \frac{1}{5} + Ce^{-t^5}}$$

274. The second-order ODE

$$y'' + y' \cdot y = 0$$

has general solution

$$y(t) = C_1 + \frac{2C_1}{C_2 e^{C_1 t} - 1}.$$

Using this, give the solution to the IVP

$$y'' + y' \cdot y = 0, \quad y(0) = 0, \quad y'(0) = 8.$$

$y(0) = C_1 + \frac{2C_1}{C_2 - 1} = (1 + \frac{2}{C_2 - 1})C_1 = 0$, so either $C_1 = 0$ or $1 + \frac{2}{C_2 - 1} = 0$. It cannot be that $C_1 = 0$ because then $y(t)$ would be 0 for all t, which would mean $y'(t) = 0$ could not satisfy $y'(0) = 8$. So it must be that $1 + \frac{2}{C_2 - 1} = 0$.

$$\begin{aligned} 1 + \frac{2}{C_2 - 1} &= 0 \\ \frac{C_2 - 1}{2} &= -1 \\ C_2 &= -1 \end{aligned}$$

With $C_2 = -1$, we have $y(t) = C_1 - \frac{2C_1}{e^{C_1 t} + 1}$. Then $y'(t) = \frac{2C_1^2 e^{C_1 t}}{(e^{C_1 t} + 1)^2}$, so $y'(0) = \frac{2C_1^2}{4} = 8$, so $C_2 = 4$. The solution to the IVP is therefore $\boxed{y(t) = 4 - \frac{8}{e^{4t} + 1}}$

275. True or false:

(a) Every first-order direct ODE is linear. True

(b) Every first-order autonomous ODE is linear. False

(c) Every first-order linear ODE is separable. False

(d) Every homogeneous first-order linear ODE is separable. True
$y' + a(t)y = 0$ is also $y' = -a(t)y$.

(e) Every homogeneous first-order linear ODE is autonomous. False

(f) Every separable ODE is either direct or autonomous. False

(g) Every first-order linear ODE is either homogeneous or separable. False

276. Complete the following table, where "CC" means constant coefficients and "hom." means homogeneous. *Note that each row must have exactly one checkmark.*

		CC hom. linear	CC non-hom. linear	non-CC hom. linear	non-CC non-hom. linear	not linear
	$y'' + 4y' = \sin(x)$	-	✔	-	-	-
	$x'' - t^2x = 0$	-	-	✔	-	-
	$x'' - tx^2 = 0$	-	-	-	-	✔
(a)	$y'' = t^2$	-	✔	-	-	-
(b)	$x'' = x^2$	-	-	-	-	✔
(c)	$x'' = t^2x$	-	-	✔	-	-
(d)	$y'' = x^2$	-	✔	-	-	-
(e)	$y'' + y' = e^t$	-	✔	-	-	-
(f)	$y'' + y' = y$	✔	-	-	-	-

277. Solve the following IVPs:

(a) $\frac{1}{2}y'' - 4y' + \frac{41}{2}y = 0, \quad y(0) = 1, \quad y'(0) = -2.$ $\boxed{y = e^{4t}\cos(5t) - \frac{6}{5}e^{4t}\sin(5t)}$

(b) $x'' + 7x' + 10x = 0, \quad x(0) = 13, \quad x'(0) = 100.$
$\boxed{x = 18e^{5t} - 5e^{-2t}}$ (see **Task 235(d)**)

(c) $4x' - 3x = 10\cos(t), \quad x(0) = 1.$ $\boxed{x = \frac{11}{5}e^{(3/4)t} + \frac{8}{5}\sin(t) - \frac{6}{5}\cos(t)}$

(d) $t\,x' + 3x = t^3, \quad x(1) = \frac{1}{2}.$ $\boxed{x = \dfrac{t^3}{6} + \dfrac{1}{3t^3}}$ (see **Task 261(e)**)

(e) $x'' - x' - 2x = 0, \quad x(0) = 5, \quad x'(0) = 8.$ $\boxed{x = \frac{13}{3}e^{2t} + \frac{2}{3}e^{-t}}$

(f) $x'' - x' + 2x = 0, \quad x(0) = 5, \quad x'(0) = 8.$ $\boxed{x = 5e^{t/2}\cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{11}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{2}t\right)}$

(g) $y'' - 12y' + 36y = 6t^2 - 1, \quad y(0) = \frac{-1}{3}, \quad y'(0) = \frac{1}{9}.$ $\boxed{y = 2e^{6t}t - \frac{1}{3}e^{6t} + \frac{1}{6}t^2 + \frac{1}{9}t}$

(h) $\begin{cases} x' = x + y, & x(0) = 5 \\ y' = 2x, & y(0) = 3 \end{cases}$ If you eliminate y, the ODE for $x(t)$ is the same as in **Task 277(e)**. $\boxed{x = \frac{13}{3}e^{2t} + \frac{2}{3}e^{-t}, \ y = \frac{13}{3}e^{2t} - \frac{4}{3}e^{-t}}$

278. One of the ODEs below is from a category that we have learned how to solve.[1] Solve that one and ignore the other two.

$$x' = x^7 \qquad \text{or} \qquad x' = t + x^7 \qquad \text{or} \qquad x'' = x^7$$

$\boxed{x' = x^7}$ is autonomous / separable, and its solution is $\boxed{x = \pm(-6t + C)^{-1/6}}$.

[1]For first-order, these include directly, autonomous, separable, and linear. For higher-order, this is only linear with constant coefficients. For systems, this is only homogeneous linear systems with constant coefficients.

279. One of the ODEs below is from a category that we have learned how to solve.[1] Solve that one and ignore the other two.

$$x'' = tx + 7 \qquad \text{or} \qquad x'' = x + 7t \qquad \text{or} \qquad x'' = 7^x + t$$

$\boxed{x'' = x + 7t}$ is non-homogeneous linear with constant coefficients, and its solution is $\boxed{x = C_1 e^t + C_2 e^{-t} - 7t}$.

280. One of the ODEs below is from a category that we have learned how to solve.[1] Solve that one and ignore the other two.

$$y''' + y = 1 \qquad \text{or} \qquad y'' + y^2 = 1 \qquad \text{or} \qquad y''' \cdot y = 1$$

$\boxed{y''' + y = 1}$ is non-homogeneous linear with constant coefficients, and its solution is $\boxed{y = C_1 e^{-t} + C_2 e^{t/2} \sin\left(\frac{\sqrt{3}}{2} t\right) + C_3 e^{t/2} \cos\left(\frac{\sqrt{3}}{2} t\right) + 1}$.