Analysis 2, Summer 2024

List 5

Review for Exam 1

123. Describe the top half of the circle $x^2 + y^2 = 12$ using parametric equations (or a single vector equation $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$) and a range of t values.

124. Calculate $\displaystyle\int_C \cos\left(\tfrac{\pi y^2}{x}\right) \mathrm{d}s$, where C is the line segment from $(0,0)$ to $(6,1)$.

125. Calculate $\displaystyle\iint_D e^{xy} \, \mathrm{d}A$ where $D = \{(x,y) : 1 \le y \le 8, 0 \le x \le \tfrac{1}{y}\}$.

126. Calculate $f'_{\hat{u}}(0,2)$ where $f(x,y) = \sin(x^2 + \pi y)$ and $\hat{u}$ is parallel to $\begin{bmatrix} \sqrt{17} \\ 8 \end{bmatrix}$.

127. Find the critical point(s) of $f(x,y) = x^2y - 5x^2 - 4xy + 20x$.

128. Find and classify the critical point(s) of $f(x,y) = \ln(-x/y) + ye^x$.

129. Find the unit vector $\hat{u} = [u_1, u_2, u_3]$ such that the rate of change of

$$f(x,y,z) = xz^2 - 11\sin(y) + x$$

at $(5,0,1)$ is as large as possible in the direction $\hat{u}$.

130. Find and classify the critical point(s) of $f(x,y) = x^2 - 10x + 13 + 4y + y^2$.

131. For $f(x,y) = x\ln(xy)$ at the point $(3, \frac{1}{3})$,
   (a) in what direction(s) is $f'_{\hat{u}}(3, \frac{1}{3})$ as large as possible?
   (b) what is the value of $f'_{\hat{u}}(3, \frac{1}{3})$ for the direction from part (a)?
   (c) in what direction(s) is $f'_{\hat{u}}(3, \frac{1}{3})$ as small (most negative) as possible?
   (d) what is the value of $f'_{\hat{u}}(3, \frac{1}{3})$ for the direction from part (c)?
   (e) in what direction(s) is $f'_{\hat{u}}(3, \frac{1}{3})$ equal to zero?

132. Let D be the region $\left\{(x,y) : -\sqrt{1-y^2} \le x \le 0\right\}$.
   (a) Draw this region.
   (b) Fill in the blanks $\displaystyle\iint_D f \, \mathrm{d}A = \int_{\square}^{\square}\int_{\square}^{\square} f \, \mathrm{d}x \, \mathrm{d}y$.
   (c) Fill in the blanks $\displaystyle\iint_D f \, \mathrm{d}A = \int_{\square}^{\square}\int_{\square}^{\square} f \, \mathrm{d}y \, \mathrm{d}x$.
   ☆(d) Fill in the blanks $\displaystyle\iint_D f \, \mathrm{d}A = \int_{\square}^{\square}\int_{\square}^{\square} f \, r \, \mathrm{d}r \, \mathrm{d}\theta$.
   ☆(e) Calculate $\displaystyle\iint_D \frac{e^{x^2+y^2}}{\pi} \, \mathrm{d}A$.

   Double integrals in "polar coordinates" are not part of MAT 1510, so you will not need to do 132(d) or 132(e) on quizzes or exams in this course.

133. Let R be the triangle with vertices $(0, 0)$, $(-6, 6)$, and $(6, 6)$, and let the density within this triangle be $\rho(x, y) = y + 1$.

(a) Evaluate $\iint_R (y+1)\,\mathrm{d}A$. (This is the mass of the triangle.)

(b) Evaluate $\iint_R (y+1)x\,\mathrm{d}A$.

(c) Evaluate $\iint_R (y+1)y\,\mathrm{d}A$.

(d) The center of mass of the triangle has coordinates $\left(\dfrac{\text{answer (b)}}{\text{answer (a)}}, \dfrac{\text{answer (c)}}{\text{answer (a)}}\right)$. Find this point.

134. For the function $f(x, y) = y\ln(x)$,

(a) Give the gradient vector ∇f at the point $(3, 6)$.

(b) Give the Hessian matrix $\mathbf{H}f$ at the point $(3, 6)$.

135. Give all (three) of the first partial derivatives and all (nine) of the second partial derivatives of

$$f(x, y, z) = z^2\ln(xy) + \cos(xz).$$

136. If $\nabla f(x, y, z) = (3x^3 + z)\hat{\imath} + ze^{yz}\hat{\jmath} + (x + ye^{yz})\hat{k}$, calculate $f''_{xx}(2, 20, -5)$.

137. If $\nabla g(x, y) = \begin{bmatrix}\sin(x)\\ \sin(y)\end{bmatrix}$, determine whether $(0, 0)$ is a local minimum, local maximum, or saddle point of $g(x, y)$.

138. For the function $f(x, y) = xy^2$ at the point $(2, 3)$,

(a) calculate the directional derivative $f'_{\hat{u}}(2, 3)$ in the direction $\hat{u} = [0, 1]$.

(b) calculate $f'_{\hat{u}}(2, 3)$ when the angle between $\hat{u}$ and $\nabla f(2, 3)$ is $60°$.

(c) give a formula using θ for the value of $f'_{\hat{u}}(2, 3)$ when the angle between $\hat{u}$ and $\nabla f(2, 3)$ is θ.

(d) what is the largest possible value of $f'_{\hat{u}}(2, 3)$, and for what unit vector $\hat{u}$ does this occur?

(e) what is the most negative possible value of $f'_{\hat{u}}(2, 3)$, and for what unit vector $\hat{u}$ does this occur?

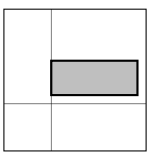
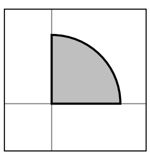
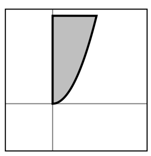
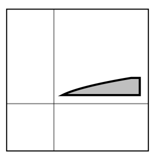
(f) give two unit vectors $\hat{u}$ for which $f'_{\hat{u}}(2, 3) = 0$.

139. Write the system of equations that would be used to find the critical point(s) of

$$f(x, y) = x\sin(xy^3).$$

(Do *not* attempt to solve the system.)

140. Which region below corresponds to $\displaystyle\int_1^3\int_{y^2}^{10} x^2\,\mathrm{d}x\mathrm{d}y$?

(A) 
(B) 
(C) 
(D) 

141. Which region above corresponds to $\int_1^3 \int_{y^2}^{10} \frac{x}{y} \, \mathrm{d}x\mathrm{d}y$?

142. For the function
$$f(x, y) = x^3 - y^x,$$
give a unit vector $\hat{u}$ for which $f'_{\hat{u}}(3, 1) = 0$.

143. Calculate $f'_{-\hat{\jmath}}(3, 7)$ for $f(x, y) = \dfrac{y}{\cos(x^x)}$.

144. If $f(x, y)$ is a function for which

$$\begin{array}{lll} f(9,-1) = 5 & f(4,7) = 6 & f(8,0) = 10 \\ f'_x(9,-1) = 0 & f'_x(4,7) = 0 & f'_x(8,0) = 0 \\ f'_y(9,-1) = 0 & f'_y(4,7) = 1 & f'_y(8,0) = 0 \\ f''_{xx}(9,-1) = \frac{1}{2} & f''_{xx}(4,7) = 2 & f''_{xx}(8,0) = -6 \\ f''_{xy}(9,-1) = \sqrt{2} & f''_{xy}(4,7) = 18 & f''_{xy}(8,0) = 0 \\ f''_{yy}(9,-1) = 12 & f''_{yy}(4,7) = 3 & f''_{yy}(8,0) = -3 \end{array}$$

(a) is $(9, -1)$ a local minimum, local maximum, saddle, or none of these?

(b) is $(4, 7)$ a local minimum, local maximum, saddle, or none of these?

(c) is $(8, 0)$ a local minimum, local maximum, saddle, or none of these?

(d) give a vector that is perpendicular to the level curve $f(x, y) = 6$ at the point $(4, 7)$.

145. Give the Hessian $\mathbf{H}f(x, y)$ of the function $f(x, y)$ for which $\nabla f = \begin{bmatrix} e^{xy}(xy+1) \\ x^2 e^{xy} \end{bmatrix}$.

☆146. Give an example of a function $f(x, y)$ for which $\nabla f = \begin{bmatrix} 2x + y^2 e^{xy^2} \\ 2xye^{xy^2} + 9y^2 \end{bmatrix}$.

147. Re-write
$$\int_0^{\sqrt{3}} \int_x^{\sqrt{3}x} \frac{y}{x^3 + y^2 x} \, \mathrm{d}y \, \mathrm{d}x \; + \int_{\sqrt{3}}^3 \int_x^3 \frac{y}{x^3 + y^2 x} \, \mathrm{d}y \, \mathrm{d}x$$
as a single iterated integral.

148. Find the value of $\iint_D \frac{25x^4}{y} \, \mathrm{d}A$ where D is the triangle with vertices $(1, 1)$ and $(4, 4)$ and $(1, 4)$.

149. Evaluate $\int_0^6 \int_{x/2}^3 \sin(\pi y^2) \, \mathrm{d}y \, \mathrm{d}x$ by reversing the order of integration (that is, by changing to an equivalent integral $\mathrm{d}x \, \mathrm{d}y$).