Analysis 2, Summer 2024

List 6

Vocabulary, direct ODEs/PDEs/IVPs

150. If $f'(x) = x$ and $f(2) = 3$, what is the function $f(x)$?

An **ordinary differential equation** (ODE) is an equation that includes a derivative of a function with one variable. The **order** of an ODE is the highest derivative that appears in the equation.

151. Give the order of each of the following differential equations:
   (a) $xy' + x^2 = 0$
   (b) $y'' = t$
   (c) $\cos(y') = \sin(y)$
   (d) $(y')^3 = e^t$
   (e) $y \cdot y'' = \sin(t)$

152. Match each ODE to its solution:

(a) $y'(x) = x + y$	(I) $y(x) = \dfrac{1}{\cos(x) - C}$
(b) $y'(x) = 2xy$	(II) $y(x) = Ce^{(x^2)}$
(c) $y'(x) = \sin(x) \cdot y^2$	(III) $y(x) = \dfrac{\pm 1}{\sqrt{2x - C}}$
(d) $y'(x) = -y^3$	(IV) $y(x) = Ce^x - x - 1$

153. Which of the following satisfy $x'' = x$?
   (a) $x = 5e^t$
   (b) $x = e^t + 9e^{-t}$
   (c) $\ln(x) = 12t$
   (d) $x = C_1e^t + C_2e^{-t}$
   (e) $2x = 19e^{-t}$
   (f) $xe^t = C_1e^{2t} + C_2$

A **partial differential equation** (PDE) is an equation that includes a partial derivative.

154. Solve the PDE $f'_x = 2x + y^2e^{xy^2}$, $f'_y = 2xye^{xy^2} + 9y^2$.

155. For each PDE below, state whether any solution $f(x, y)$ exists.
   (a) $f'_x = 5, f'_y = 6$
   (b) $f'_x = 5x, f'_y = 6y$
   (c) $f'_x = 5y, f'_y = 6x$
   (d) $f'_x = 5y, f'_y = 5x$

156. Give solutions for the PDEs from Task 155 that have solutions.

A **general solution** to an ODE describes all possible solutions to that differential equation, using one of more "constants of integration" (usually C; or C_1, C_2, etc.). A **particular solution** is a solution that does *not* use arbitrary constants.

157. Give three different particular solutions to $y' = 11x^4$.

158. Give the general solution to $y' = \sin(x)$.

159. Solve the ODE $y' = 5x + e^x$. (That is, find its general solution).

160. Solve the ODE $x' = e^{3t}$.

161. Solve the ODE $y' = \dfrac{x^4}{\sqrt{x^5+1}}$.

An **initial value** (IV) or **initial condition** (IC) is a piece of information giving the value of a function or its derivative for a particular value of the input variable. An **initial value problem** (IVP) is a differential equation along with one or more initial values.

The general solution to an IVP ignores the initial condition and describes all solutions to the differential equation only.

162. Solve the IVP $y' = 4x^2 - 9,\ y(0) = 2$.

163. Solve the IVPs:
   (a) $x' = t,\ x(2) = 3$.
   (b) $y' = t,\ y(2) = 3$.
   (c) $y' = 5x + e^x,\ y(0) = 14$.
   (d) $y' = 5x + e^x,\ y(2) = 14$.

164. Solve the IVP
$$\frac{\partial f}{\partial x} = \frac{6y}{x+1}, \quad \frac{\partial f}{\partial y} = 8e^y + 6\ln(x+1), \quad f(0,0) = 12.$$

165. The first-order ODE $y'(x) = y(x)$ has general solution $y(x) = Ce^x$. Using this, give the solution to the IVP $y'(x) = y(x),\ y(0) = 16$.

166. The first-order ODE
$$x' = t^2x^2$$
has general solution
$$x = \frac{-3}{t^3 + C}.$$
Using this, give the solution to the IVP
$$x' = t^2x^2, \quad x(1) = \tfrac{1}{2}.$$

167. The second-order ODE
$$y'' - 3y' - 10y = -40$$
has general solution
$$y = 4 + C_1e^{-2x} + C_2e^{5x}.$$
Using this, give the solution to the IVP
$$y'' - 3y' - 10y = -40, \qquad y(0) = 15, \quad y'(0) = -8.$$