List 2
Tangent lines, monotonicity, critical points

49. Give the slope of the tangent line to $y = 3x + \dfrac{7}{x}$ at $x = 2$.

50. Give an equation for the tangent line to $y = 3x + \dfrac{7}{x}$ at $x = 2$.

51. Give an equation for the tangent line to $y = \sin(x)$ at $x = \frac{\pi}{3}$.

☆52. Find a number k so that the tangent line to $y = x^2 + 4x$ at $x = k$ and the tangent line to $y = \frac{1}{5}x^5 - 8x + 1$ at $x = k$ are parallel.

53. Use the fact that
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sin(\tfrac{1}{x})\right] = \frac{-\cos(\tfrac{1}{x})}{x^2}$$
to find an equation for the tangent line to $y = \sin(\frac{1}{x})$ at $x = \frac{1}{\pi}$.

54. (a) For what value(s) of x does $x^3 - 18x^2 = 0$?

    (b) For what value(s) of x does $3x^2 - 36x = 0$?

    (c) For what value(s) of x does $6x - 36 = 0$?

55. At what values of x is the tangent line to $y = x^3 - 18x^2$ horizontal?

A number c in the domain of $f(x)$ is a **critical point** of $f(x)$ if either $f'(c) = 0$ or $f'(c)$ does not exist.

If $f'(a) > 0$ then f is **increasing** at $x = a$.
If $f'(a) < 0$ then f is **decreasing** at $x = a$.

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56. What are the critical points of $x^3 - 18x^2$?

57. Find all the critical points of $8x^5 - 57x^4 - 24x^3 + 9$.

58. List all the critical points of the function graphed below (portions of its tangent lines at $x = -2$, $x = 1$, $x = 3$, and $x = 6$ are shown as dashed lines).
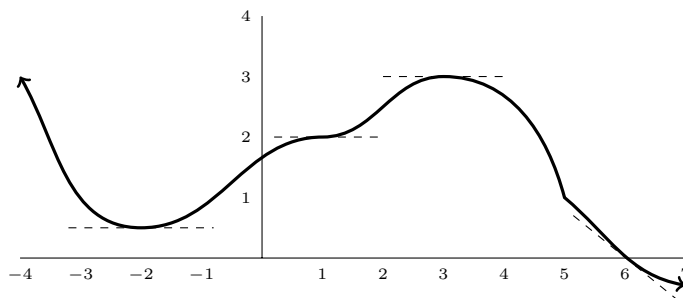
59. Is the function
$$f(x) = x^8 - 6x^3 + 29x - 12$$
increasing, decreasing, or neither when $x = -1$?

60. (a) On what (possibly infinite) interval or intervals is $2x^3 - 3x^2 - 12x$ increasing?

(b) On what (possibly infinite) interval or intervals is $2x^3 - 3x^2 - 12x$ decreasing?

61. Suppose $f(x)$ is a function that is increasing when $x = 5$.

(a) Is it possible to know the sign of $f(5)$? (That is, it is possible to know which of $f(5) > 0$ or $f(5) = 0$ or $f(5) < 0$ is true?)

(b) Is it possible to know the sign of $f'(5)$?

(c) Is it possible to know the sign of $f''(5)$?

62. On what interval(s) is $x^2 - 8\sqrt{x} + 7$ decreasing?

63. List all critical points of $f(x) = \frac{3}{4}x^4 - 7x^3 + 15x^2$ in the interval $[-3, 3]$.

64. For each graph below, is there a critical point at $x = 0$?
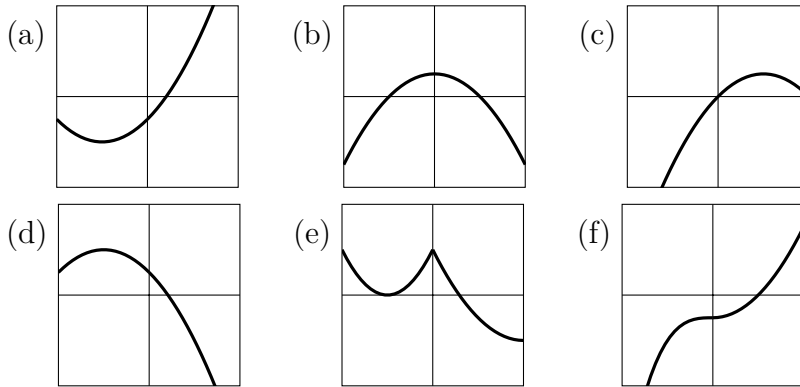
65. The derivative of

$$f(x) = \frac{4x + 1}{3x^2 - 12} \qquad \text{is} \qquad f'(x) = \frac{-4x^2 - 2x - 16}{3x^4 - 24x^2 + 48}.$$

Using this, find all the critical points of $f(x)$.

66. Find all the critical points of

(a) $f(x) = x^2 - \cos(x)$.

(b) $f(x) = 2x + \cos(x)$.

(c) $f(x) = x + 2\cos(x)$.

(d) $f(x) = x^2 + x - \sin(x)$.

☆ (e) $f(x) = x^2 + x + \cos(x)$.

67. Match the functions (a)-(f) to their derivatives (I)-(VI).

(a)

(I)

(b)

(II)

(c)

(III)

(d)

(IV)

(e)

(V)

(f)

(VI)