Analysis 1, Summer 2024

List 4

Limits

93. Calculate the following limits:

(a) $\displaystyle\lim_{x\to 7} \frac{x^2-4x-21}{x^2-11x+28}$

(b) $\displaystyle\lim_{x\to 0} \frac{x^3-8x^2+3x+5}{x^9-6x^5+x^4-12x+1}$

(c) $\displaystyle\lim_{x\to\infty} \frac{3x^3-2x+1}{6x^3+x^2+x+19}$

(d) $\displaystyle\lim_{x\to\infty} \frac{3x^2-2x+1}{6x^3+x^2+x+19}$

(e) $\displaystyle\lim_{x\to 0} \left(\frac{8x-1}{x-x^2}+\frac{1}{x}\right)$

(f) $\displaystyle\lim_{n\to\infty} \left(\sqrt{9n^2+5n}-3n\right)$

(g) $\displaystyle\lim_{x\to\infty} \left(\sqrt{9x^2+5x}-3x\right)$

(h) $\displaystyle\lim_{x\to\infty} (4^x+1)^{1/4}$

☆(i) $\displaystyle\lim_{x\to\infty} (4^x+x)^{1/x}$

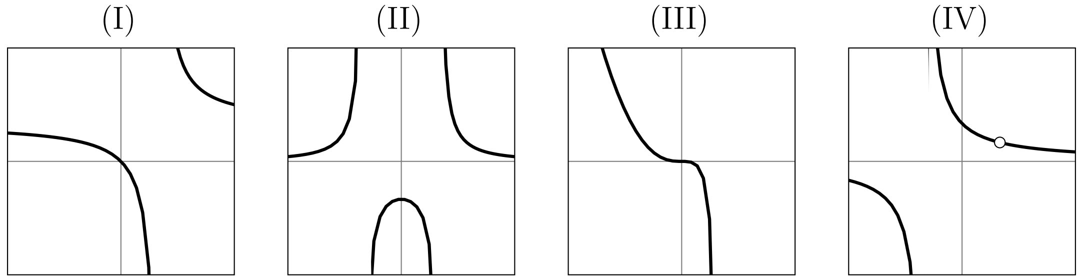
94. At $x=1$, does the function

$$f(x)=\begin{cases} 2x-1 & \text{if } x\le 1,\\ x/3 & \text{if } 1<x<9,\\ \sqrt{x} & \text{if } x\ge 9\end{cases}$$

have a jump, hole, vertical asymptote, or none of these? At $x=9$?

95. Match the functions with their graphs:

(a) $\dfrac{1}{x^2-1}$ (c) $\dfrac{x-1}{x^2-1}$ (b) $\dfrac{x^3}{x-1}$ (d) $\dfrac{x}{x-1}$



96. Suppose $\displaystyle\lim_{x\to 10^-} f(x)=2$.

(a) If the graph of f has a hole at $x=10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} f(x)$ from only this information?

(b) If the graph of f has a hole at $x=10$, is it possible to know the value of $f(2)$ from only this information?

(c) If the graph of f has a jump at $x=10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} f(x)$ from only this information?

(d) If the graph of f has a vertical asymptote at $x=10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} f(x)$ from only this information?

(e) If the graph of f has a vertical asymptote at $x=10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} |f(x)|$ from only this information?

97. Find the following limits, if they exist:

(a) $\lim\limits_{x\to 0^+} \dfrac{|x|}{x}$ (b) $\lim\limits_{x\to 0^-} \dfrac{|x|}{x}$ (c) $\lim\limits_{x\to 0} \dfrac{|x|}{x}$

98. For which value(s) of the parameter a does the function

$$f(x) = \frac{x^2 - a}{x^2 + a}$$

have a vertical asymptote at $x = 3$?

99. For which value(s) of the parameter a is the function from Task 98 continuous?

100. For which value(s) of the parameter a is the function

$$f(x) = \begin{cases} x - 6a & \text{if } x < a \\ ax + 4 & \text{if } x \geq a \end{cases}$$

continuous?

101. Which limit expression below gives the derivative of x^3 at the point $x = 2$?

(A) $\lim\limits_{x\to 2} \dfrac{x^3 - 8}{x}$ (C) $\lim\limits_{h\to 0} \dfrac{(2+h)^3 - 8}{h}$

(B) $\lim\limits_{h\to 0} \dfrac{h^3 - 8}{h}$ (D) $\lim\limits_{h\to 0} \dfrac{(2+h)^3 - h^3}{h}$

L'Hôpital's Rule: if $\lim\limits_{x\to a} f(x) = 0$ and $\lim\limits_{x\to a} g(x) = 0$ and $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ exists, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

The same substitution works if $\lim\limits_{x\to a} f(x) = \infty$ or $-\infty$ and $\lim\limits_{x\to a} g(x) = \infty$ or $-\infty$. And also for one-sided limits and for $x \to \infty$ and $x \to -\infty$.

102. Calculate $\lim\limits_{x\to 1} \dfrac{3x^3 + 4x^2 - 13x + 6}{2x^4 + x^3 - x^2 + x - 3}$.

103. (a) Find $\lim\limits_{x\to 1} \dfrac{x^2 - 4x + 18}{3x^2 + 10}$. (b) Find $\lim\limits_{x\to 1} \dfrac{2x - 4}{6x}$.

(c) Why are the answers to (a) and (b) not equal?

104. Find $\lim\limits_{x\to 0} \dfrac{2\sin(x) - \sin(2x)}{x - \sin(x)}$.

$$(e^x)' = e^x \qquad\qquad \big(\ln(x)\big)' = \frac{1}{x}$$

105. For the function $f(x) = x^2 e^{-x}$, find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to -\infty} f(x)$.

106. Calculate $\lim\limits_{x\to 4} \dfrac{\sin(\pi x)}{\ln(x - 3)}$.

107. Give an equation for the tangent line to $y = e^{3x}(\cos(4x))^5$ at $x = 0$.

108. Is $y = e^{\sin(x)}$ concave up or concave down when $x = \pi$?

109. Find the local extremes of $f(x) = \sqrt{x} - \ln(x)$.

110. Find the limit $\displaystyle\lim_{x\to 0^+} \frac{\ln(x)}{1/x}$.

111. Find the limit $\displaystyle\lim_{x\to 0^+} x\ln(x)$

112. (a) Calculate $\lim\limits_{n\to\infty} n \cdot \ln\left(1 + \frac{1}{n}\right)$ using algebra and L'Hôpital.

(b) Calculate $\lim\limits_{n\to\infty} \ln\left(\left(1 + \frac{1}{n}\right)^n\right)$ using log rules and L'Hôpital.

(c) Calculate $\displaystyle\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ using the fact that $x = e^{\ln(x)}$ and therefore

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e^{\ln\left(\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right)}.$$