Analysis 1, Summer 2024

List 6

Review for Exam 2

161. Give an equation for the line through the point $(5, 17)$ with slope $\frac{1}{3}$.

162. Give an equation for the tangent line to $y = \sqrt{x} + x^3$ at $x = 1$.

163. Give an equation for the tangent line to $y = e^{4x \cos x}$ at $x = 0$.

164. Use the Quotient Rule and the Product Rule to compute $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for $y = \dfrac{\ln(x)e^x}{x^2}$.

165. Calculate the derivative $\ln(5x)$ in two ways:
   (a) Use the rule $\frac{\mathrm{d}}{\mathrm{d}x}\big[\ln(x)\big] = \frac{1}{x}$ along with the Chain Rule (here $\ln(5x) = f(g(x))$ with $f(x) = \ln(x)$ and $g(x) = 5x$).
   (b) Use algebra to rewrite $\ln(5x) = \ln(x) + \ln(5)$ and then find the derivative of that function.

166. On what interval(s) is the function $x^3 - 6x + 11$ increasing? On what interval(s) is it concave up?

167. Find the x-coordinates of all inflection points of $x^4 + 9x^3 - 15x^2 + 17$.

168. Find the x-coordinates of all inflection points of

$$f(x) = \tfrac{3}{10}x^5 - 5x^4 + 32x^3 - 96x^2 + 28.$$

169. If f is a smooth function with

x	-2	-1	0	1	2	3	4
f	3	5	-3	7	8	9	12
f'	2	0	-1	-1	1	3	0
f''	0	4	1	-1	$\frac{-8}{3}$	0	1

answer the following:
   (a) Does f have a critical point at $x = 0$?
   (b) Does f have a local minimum at $x = -1$?
   (c) Does f have a local maximum at $x = 4$?
   (d) It it possible that f has an absolute minimum at $x = -1$?
   (e) It it possible that f has an absolute maximum at $x = -1$?
   (f) It it possible that f has an inflection point at $x = 3$?
   (g) It it possible that f has an inflection point at $x = 4$?

170. Find all the critical point(s) of the function

$$f(x) = x^4 - 12x^3 + 30x^2 - 28x$$

and classify each one as a local minimum, local maximum, or neither.

171. Find all the critical point(s) of the function

$$f(x) = x(6 - x)^{2/3}$$

and classify each one as a local minimum, local maximum, or neither.

172. Calculate the following limits, if they exist:

(a) $\lim_{x\to 4} \dfrac{x^2 - x - 12}{x^2 - 2x - 8}$

(b) $\lim_{x\to 4} \dfrac{x^2 + x - 12}{x^2 - 2x - 8}$

(c) $\lim_{x\to\infty} xe^{-x}$

(d) $\lim_{x\to 0+} x^2 \ln(x)$

(e) $\lim_{x\to 1} x^2 \ln(x)$

173. Compute $\lim_{x\to 0}(\cos 6x)^{1/x^2}$. Hint: First compute $\lim_{x\to 0} \ln\Big((\cos 6x)^{1/x^2}\Big)$.

174. Compute the following indefinite integrals:

(a) $\int 6\,\mathrm{d}x$

(b) $\int (2x+6)\,\mathrm{d}x$

(c) $\int \frac{8}{x}\,\mathrm{d}x$

(d) $\int \frac{8}{q}\,\mathrm{d}q$

(e) $\int x^2\cos(x^3)\,\mathrm{d}x$

(f) $\int x^2\cos(x)\,\mathrm{d}x$

175. Compute the following definite integrals:

(a) $\int_1^5 (2x+6)\,\mathrm{d}x$

(b) $\int_0^\pi \tfrac{1}{3}\sin(u)\,\mathrm{d}u$

(c) $\int_1^4 (x^3 + 2x - 7)\,\mathrm{d}x$

(d) $\int_0^\pi 2e^t\sin(5t)\,\mathrm{d}t$

176. Find the value of $\int_{-2}^{2} \sqrt{4 - x^2}\,\mathrm{d}x$.

177. Compute the following integrals of rational functions:

(a) $\int \dfrac{2x+3}{10x^2 + 30x + 40}\,\mathrm{d}x$

(b) $\int \dfrac{10x^2 + 30x + 40}{5x}\,\mathrm{d}x$

(c) $\int_1^3 \dfrac{10x^2 + 30x + 40}{5x}\,\mathrm{d}x$

☆(d) $\int \dfrac{3}{10x^2 + 40}\,\mathrm{d}x$

178. Find the area of the domain

$$\{(x,y) : 0 \le x \le \pi,\ 0 \le y \le 5\sin(\tfrac{x}{2})\}.$$

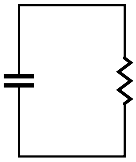
179. Find the area of the domain

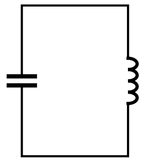
$$\{(x,y) : 0 \le x \le \pi,\ 0 \le y \le 2x\sin(3x) + 4x\}.$$

180. Find the area of the region bounded by the curves $y = x^2$ and $y = 10 - x^2$.

181. Calculate the area of the region bounded by $x = 1$, $y = 1$, and $y = \ln(x)$.

☆182. Below are two circuits and an equation involving a derivative for each of them. Here R (resistance), C (capacitance), and L (inductance) are all constants, but the voltage $V = V(t)$ is a function of time.


$$V + RC\frac{\mathrm{d}V}{\mathrm{d}t} = 0$$


$$V + LC\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 0$$

(a) Could the first circuit have $V(t) = 5e^{-t/12}$? The second circuit?

(b) Could the first circuit have $V(t) = 5(1 - e^{-t/12})$? The second circuit?

(c) Could the first circuit have $V(t) = 5\sin(2\pi t)$? The second circuit?

(d) Could the first circuit have $V(t) = 0$? The second circuit?

☆183. The "information entropy" h of a weighted coin where Heads (Orzel) has probability p_1 and Tails (Reszka) has probability p_2 is

$$-p_1 \ln(p_1) - p_2 \ln(p_2).$$

Labeling $x = p_1$, we have $p_2 = 1 - x$ (because probabilities must add to 1), so the entropy of a coin with $P(\text{Heads}) = x$ is

$$h(x) \quad = \quad -x\ln(x) - (1-x)\ln(1-x).$$

Determine the x-value that gives the maximum of $h(x)$.
(This will be the *most random* kind of coin.)

184. Give the one hundredth derivative of $x\,e^x$, that is, $\dfrac{\mathrm{d}^{100}}{\mathrm{d}x^{100}}\big[x\,e^x\big]$.

185. For the function

$$f(x) = xe^{-x/4},$$

(a) Give the equation for the tangent line to $y = f(x)$ at $x = -4$.

(b) Compute the limits $\lim_{x\to 1} f(x)$ and $\lim_{x\to\infty} f(x)$.

(c) Find the critical point of $f(x)$. (There is only one.)

(d) Is the point from part (c) a local minimum, local maximum, or neither?

(e) Find the inflection point of $f(x)$. (There is only one.)

(f) Calculate the area of the domain $\{(x, y) : 0 \le x \le 4, 0 \le y \le f(x)\}$.