Analysis 1, Summer 2023

List 10

Review for Exam 2

240. Calculate the following limits, if they exist:

(a) $\lim_{x\to 4} \dfrac{x^2 - x - 12}{x^2 - 2x - 8}$

(b) $\lim_{x\to 4} \dfrac{x^2 + x - 12}{x^2 - 2x - 8}$

(c) $\lim_{x\to\infty} xe^{-x}$

(d) $\lim_{x\to 0^+} x^2 \ln(x)$

(e) $\lim_{x\to 1} x^2 \ln(x)$

241. Compute $\lim_{x\to 0} \dfrac{2e^x - x^2 - 2x - 2}{x^3}$.

242. Compute $\lim_{x\to 0}(\cos 6x)^{1/x^2}$. Hint: First compute $\lim_{x\to 0} \ln\Big((\cos 6x)^{1/x^2}\Big)$.

243. Give an equation for the tangent line to $y = \sqrt{x} + x^3$ at $x = 1$.

244. Use the Quotient Rule and the Product Rule to compute $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for $y = \dfrac{\ln(x)e^x}{x^2}$.

245. Give an equation for the tangent line to $y = e^{4x\cos x}$ at $x = 0$.

246. Calculate the derivative of e^{5x} in two ways:

(a) Use the rule $\frac{\mathrm{d}}{\mathrm{d}x}\big[e^x\big] = e^x$ along with the Chain Rule (here $e^{5x} = f(g(x))$ with $f(x) = e^x$ and $g(x) = 5x$).

(b) Use algebra to rewrite $e^{5x} = (e^5)^x$ and then find the derivative of that function using the rule $\frac{\mathrm{d}}{\mathrm{d}x}\big[a^x\big] = a^x \cdot \ln(a)$.

247. Calculate the derivative $\ln(5x)$ in two ways:

(a) Use the rule $\frac{\mathrm{d}}{\mathrm{d}x}\big[\ln(x)\big] = \frac{1}{x}$ along with the Chain Rule (here $\ln(5x) = f(g(x))$ with $f(x) = \ln(x)$ and $g(x) = 5x$).

(b) Use algebra to rewrite $\ln(5x) = \ln(x) + \ln(5)$ and then find the derivative of that function.

248. On what interval(s) is the function $x^3 - 6x + 11$ increasing?

249. On what interval(s) is the function $x^3 - 6x + 11$ concave up?

250. Find the x-coordinates of all critical points of $(2x + 3)e^{4x}$.

251. Find the x-coordinates of all inflection points of $x^4 + 9x^3 - 15x^2 + 17$.

252. Find the x-coordinates of all inflection points of $x^5 + 10x^4 - 50x^3 + 80x^2 - 15$.

253. Find the absolute minimum of $f(x) = \frac{1}{4}x^4 - 4x^3 + 22x^2 - 48x + 32$ on $[1, 9]$.

254. Find the critical point(s) of $g(x) = \sqrt[3]{3x^2 + 4x + 1}$.

255. Find all the critical point(s) of the function

$$f(x) = x^4 - 12x^3 + 30x^2 - 28x$$

and classify each one as a local minimum, local maximum, or neither.

256. Find all the critical point(s) of the function

$$f(x) = x(6-x)^{2/3}$$

and classify each one as a local minimum, local maximum, or neither.

☆257. Suppose $f(x)$ is a differentiable function for which $f(6) = 2$ and $f'(6) = 0$ and $f''(6) = 3$. Does the function have a local minimum at $x = 6$? A local maximum?

☆258. Suppose $f(x)$ is a differentiable function for which $f(3) = 0$ and $f'(3) = 2$ and $f''(3) = 6$. Does the function have a local minimum at $x = 3$? A local maximum?

259. Calculate the value of $\int_{-2}^{2} (4 - x^2)\,\mathrm{d}x$.

260. Find the value of $\int_{-2}^{2} \sqrt{4 - x^2}\,\mathrm{d}x$.

261. Compute the following indefinite integrals:

(a) $\int 6\,\mathrm{d}x$

(b) $\int (2x + 6)\,\mathrm{d}x$

(c) $\int \frac{8}{x}\,\mathrm{d}x$

(d) $\int \frac{8}{q}\,\mathrm{d}q$

(e) $\int x^2 \cos(x^3)\,\mathrm{d}x$

(f) $\int x^2 \cos(x)\,\mathrm{d}x$

262. Compute the following definite integrals:

(a) $\int_1^5 (2x + 6)\,\mathrm{d}x$

(b) $\int_0^\pi \frac{1}{3}\sin(u)\,\mathrm{d}u$

(c) $\int_1^4 (x^3 + 2x - 7)\,\mathrm{d}x$

(d) $\int_0^\pi 2e^t \sin(5t)\,\mathrm{d}t$

263. Compute the following integrals of rational functions:

(a) $\int \frac{2x + 3}{10x^2 + 30x + 40}\,\mathrm{d}x$

(b) $\int \frac{10x^2 + 30x + 40}{5x}\,\mathrm{d}x$

(c) $\displaystyle\int_1^3 \frac{10x^2+30x+40}{5x}\,\mathrm{d}x$

(d) $\displaystyle\int \frac{3}{10x^2+40}\,\mathrm{d}x$

(e) $\displaystyle\int_0^2 \frac{3}{10x^2+40}\,\mathrm{d}x$

(f) $\displaystyle\int_2^\infty \frac{1}{x^5}\,\mathrm{d}x$

264. Find the area of the domain

$$\{(x,y) : 0 \le x \le \pi,\ 0 \le y \le 5\sin(\tfrac{x}{2})\}.$$

265. Find the area of the domain

$$\{(x,y) : 0 \le x \le \pi,\ 0 \le y \le 2x\sin(3x)+4x\}.$$

266. Find the area of the region bounded by the curves $y = x^2$ and $y = 10 - x^2$.

267. Calculate the area of the region bounded by $x = 1$, $y = 1$, and $y = \ln(x)$.

☆268. (a) Find the area of the region bounded by $y = x^2 + a$ and $y = ax^2 + 2$, where $a \in [0, 1)$ is a parameter (your answer will be a formula using a).

(b) Among all such shapes, what is the smallest possible area?

269. Calculate the volume of the solid formed by rotating

$$\{(x,y) : 0 \le x \le \pi,\ 0 \le y \le x\sqrt{\sin x}\}$$

around the x-axis.

270. Calculate the volume of the solid formed by rotating the region from Task 266 around the y-axis.

271. Find the volume of the solid formed by rotating the region bounded by $y = -x^2 + 10x - 21$ and the x-axis around the x-axis.