Analysis 1, Summer 2023

List 4

Review for midterm exam

101. Calculate the following limits:

(a) $\displaystyle\lim_{x\to\infty} \frac{3x^3-2x+1}{6x^3+x^2+x+19} = \boxed{\frac{1}{2}}$

(b) $\displaystyle\lim_{x\to\infty} \frac{3x^2-2x+1}{6x^3+x^2+x+19} = \boxed{0}$

(c) $\displaystyle\lim_{x\to 0} \left(\frac{8x-1}{x-x^2}+\frac{1}{x}\right) = \boxed{7}$

(d) $\displaystyle\lim_{n\to\infty} \left(\sqrt{9n^2+5n}-3n\right) = \boxed{\frac{5}{6}}$

(e) $\displaystyle\lim_{n\to\infty} (4^n+1)^{1/4} = \boxed{\infty}$

(f) $\displaystyle\lim_{n\to\infty} (4^n+n)^{1/n} = \boxed{4}$

(g) $\displaystyle\lim_{x\to 7} \frac{x^2-4x-21}{x^2-11x+28} = \boxed{\frac{10}{3}}$

(h) $\displaystyle\lim_{x\to 0} \frac{x^3-8x^2+3x+5}{x^9-6x^5+x^4-12x+1} = \boxed{5}$

102. Suppose $\displaystyle\lim_{x\to 10^-} f(x) = 2$.

(a) If the graph of f has a hole at $x = 10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} f(x)$ from only this information? $\boxed{\text{Yes: } 2}$

(b) If the graph of f has a hole at $x = 10$, is it possible to know the value of $f(2)$ from only this information? $\boxed{\text{No}}$

(c) If the graph of f has a jump at $x = 10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} f(x)$ from only this information? $\boxed{\text{No}}$

(d) If the graph of f has a vertical asymptote at $x = 10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} f(x)$ from only this information?

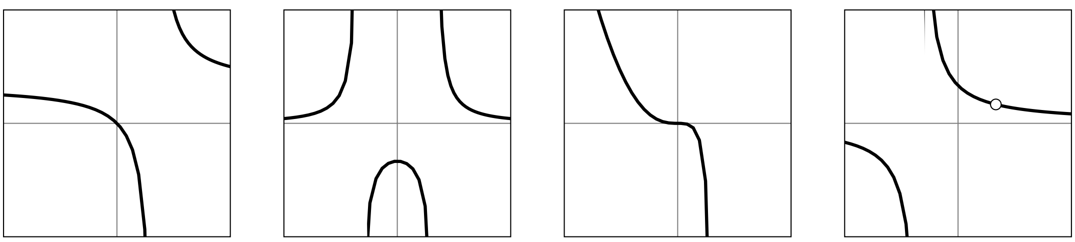
$\boxed{\text{No}}$, but it must be either $+\infty$ or $-\infty$.

(e) If the graph of f has a vertical asymptote at $x = 10$, is it possible to know the value of $\displaystyle\lim_{x\to 10^+} |f(x)|$ from only this information? $\boxed{\text{Yes: } \infty}$

103. Match the functions with their graphs:

(a) $\dfrac{1}{x^2-1}$ $\boxed{\text{(II)}}$ (c) $\dfrac{x-1}{x^2-1}$ $\boxed{\text{(IV)}}$ (b) $\dfrac{x^3}{x-1}$ $\boxed{\text{(III)}}$ (d) $\dfrac{x}{x-1}$ $\boxed{\text{(I)}}$

(I) (II) (III) (IV)



104. At $x = 9$, does the function

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \le 1, \\ \log_3(x) & \text{if } 1 < x < 9, \\ \sqrt{x} & \text{if } x \ge 9 \end{cases}$$

have a jump, hole, vertical asymptote, or none of these?
$\boxed{\text{Jump}}$ because $\lim_{x \to 9^-} f(x) = \log_3(9) = 2$ does not equal $\lim_{x \to 9^+} f(x) = \sqrt{9} = 3$.

105. For which value(s) of the parameter a does the function

$$f(x) = \frac{x^2 - a}{x^2 + a}$$

have a vertical asymptote at $x = 2$? $\boxed{a = -4}$

106. For which value(s) of the parameter a is the function from Task 105 continuous?
any $\boxed{a > 0}$

107. Which limit expression below gives the derivative of x^3 at the point $x = 2$? $\boxed{\text{(C)}}$

(A) $\lim_{x \to 2} \frac{x^3 - 8}{x}$ (C) $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

(B) $\lim_{h \to 0} \frac{h^3 - 8}{h}$ (D) $\lim_{h \to 0} \frac{(2+h)^3 - h^3}{h}$

108. (a) Find $\left(x^{10} + 100x + 1000\right)' = \boxed{10x^9 + 100}$

(b) Find $D\left[9x + \sqrt{9x}\right] = D\left[9x + 3x^{1/2}\right] = \boxed{9 + \frac{3}{2}x^{1/2}}$

(c) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[(2x + 3)^2\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[4x^2 + 12x + 9\right] = \boxed{8x + 12}$

(d) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for $y = \frac{x + 12}{2x}$. $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x+12}{2x}\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{2} + \frac{6}{x}\right] = 0 + \boxed{\frac{-6}{x^2}}$

109. Calculate $f'(2)$ for the function $f(x) = x^4 + 4x$. $\boxed{36}$

110. Find the *slope* of the tangent line to $y = x^4 + 4x$ at the point $(2, 24)$. This is exactly the same as Task 109! $\boxed{36}$

111. Give an *equation* for the tangent line to $y = x^4 + 4x$ through the point $(2, 24)$. The line through $(2, 24)$ with slope 36 can be described by $\boxed{y = 24 + 36(x - 2)}$ or by $\boxed{y = 36x - 48}$ or other formats.

112. Give an equation for the tangent line to $y = \frac{1}{\sqrt{x}}$ at $x = 4$. $y = x^{-1/2}$, so $y' = \frac{-1}{2}x^{-3/2}$, so the slope is $y'(4) = \frac{-1}{2}(4)^{-3/2} \frac{-1}{2}(2)^{-3} = \frac{-1}{2} \cdot \frac{1}{8} = \frac{-1}{16}$. The line through $(4, \frac{1}{2})$ with slope $\frac{-1}{16}$ is $\boxed{y = \frac{1}{2} - \frac{1}{16}(x - 4)}$ or $\boxed{y = \frac{-1}{16}x + \frac{3}{4}}$.