

List 4

Review for midterm exam

101. Calculate the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{3x^3 - 2x + 1}{6x^3 + x^2 + x + 19}$

(b) $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 1}{6x^3 + x^2 + x + 19}$

(c) $\lim_{x \rightarrow 0} \left(\frac{8x - 1}{x - x^2} + \frac{1}{x} \right)$

(d) $\lim_{n \rightarrow \infty} (\sqrt{9n^2 + 5n} - 3n)$

(e) $\lim_{n \rightarrow \infty} (4^n + 1)^{1/4}$

(f) $\lim_{n \rightarrow \infty} (4^n + n)^{1/n}$

(g) $\lim_{x \rightarrow 7} \frac{x^2 - 4x - 21}{x^2 - 11x + 28}$

(h) $\lim_{x \rightarrow 0} \frac{x^3 - 8x^2 + 3x + 5}{x^9 - 6x^5 + x^4 - 12x + 1}$

102. Suppose $\lim_{x \rightarrow 10^-} f(x) = 2$.

(a) If the graph of f has a hole at $x = 10$, is it possible to know the value of $\lim_{x \rightarrow 10^+} f(x)$ from only this information?

(b) If the graph of f has a hole at $x = 10$, is it possible to know the value of $f(2)$ from only this information?

(c) If the graph of f has a jump at $x = 10$, is it possible to know the value of $\lim_{x \rightarrow 10^+} f(x)$ from only this information?

(d) If the graph of f has a vertical asymptote at $x = 10$, is it possible to know the value of $\lim_{x \rightarrow 10^+} f(x)$ from only this information?

(e) If the graph of f has a vertical asymptote at $x = 10$, is it possible to know the value of $\lim_{x \rightarrow 10^+} |f(x)|$ from only this information?

103. Match the functions with their graphs:

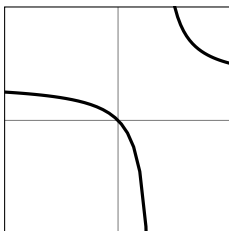
(a) $\frac{1}{x^2 - 1}$

(c) $\frac{x - 1}{x^2 - 1}$

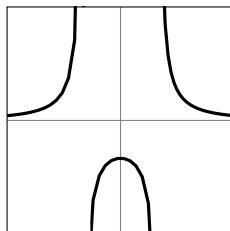
(b) $\frac{x^3}{x - 1}$

(d) $\frac{x}{x - 1}$

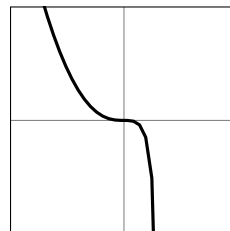
(I)



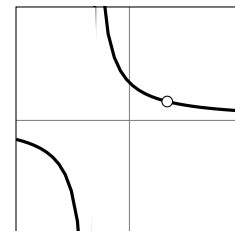
(II)



(III)



(IV)



104. At $x = 9$, does the function

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 1, \\ \log_3(x) & \text{if } 1 < x < 9, \\ \sqrt{x} & \text{if } x \geq 9 \end{cases}$$

have a jump, hole, vertical asymptote, or none of these?

105. For which value(s) of the parameter a does the function

$$f(x) = \frac{x^2 - a}{x^2 + a}$$

have a vertical asymptote at $x = 2$?

106. For which value(s) of the parameter a is the function from Task 105 continuous?

107. Which limit expression below gives the derivative of x^3 at the point $x = 2$?

(A) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x}$

(C) $\lim_{h \rightarrow 0} \frac{(2 + h)^3 - 8}{h}$

(B) $\lim_{h \rightarrow 0} \frac{h^3 - 8}{h}$

(D) $\lim_{h \rightarrow 0} \frac{(2 + h)^3 - h^3}{h}$

108. (a) Find $(x^{10} + 100x + 1000)'$.

(b) Find $D[9x + \sqrt{9x}]$.

(c) Find $\frac{d}{dx} [(2x + 3)^2]$.

(d) Find $\frac{dy}{dx}$ for $y = \frac{x + 12}{2x}$.

109. Calculate $f'(2)$ for the function $f(x) = x^4 + 4x$.

110. Find the *slope* of the tangent line to $y = x^4 + 4x$ at the point $(2, 24)$.

111. Give an *equation* for the tangent line to $y = x^4 + 4x$ through the point $(2, 24)$.

112. Give an equation for the tangent line to $y = \frac{1}{\sqrt{x}}$ at $x = 4$.