Analysis 1, Summer 2023

List 9

Integration by parts, area, volume of revolution

220. Fill in the missing parts of the table:

$f =$	$\sin(x)$	$\ln(x)$	x^3			
$\mathrm{d}f =$	$\cos(x)\,\mathrm{d}x$			$x\,\mathrm{d}x$	$\frac{\mathrm{d}x}{x}$	$\sin(x)\,\mathrm{d}x$

221. Find the derivative of $2xe^{2x}$.

Integration by parts for indefinite integrals:

$$\int u\,dv = uv - \int v\,\mathrm{d}u.$$

222. Use integration by parts with $u = 4x$ and $\mathrm{d}v = e^{2x}\,\mathrm{d}x$ to evaluate $\int 4xe^{2x}\,\mathrm{d}x$.

223. Use integration by parts with $u = \ln(x)$ and $\mathrm{d}v = 1\,\mathrm{d}x$ to find $\int \ln(x)\,\mathrm{d}x$.

224. Find the following indefinite integrals using integration by parts:

(a) $\int x\sin(x)\,\mathrm{d}x$

(b) $\int x\cos(8x)\,\mathrm{d}x$

(c) $\int \frac{\ln(x)}{x^5}\,\mathrm{d}x$

(d) $\int x^2\cos(4x)\,\mathrm{d}x$

(e) $\int (4x+12)e^{x/3}\,\mathrm{d}x$

(f) $\int \cos(x)e^{2x}\,\mathrm{d}x$

225. Calculate the following definite integrals using integration by parts:

(a) $\int_0^6 (4x+12)e^{x/3}\,\mathrm{d}x$

(b) $\int_1^2 x\ln(x)\,\mathrm{d}x$

(c) $\int_0^1 t\sin(\pi t)\,\mathrm{d}t$

(d) $\int_0^\pi x^4\cos(4x)\,\mathrm{d}x$

☆226. Prove that $\int_1^\pi \ln(x)\cos(x)\,\mathrm{d}x = \int_1^\pi \frac{-\sin(x)}{x}\,\mathrm{d}x$.

☆227. If $g(0) = 8$, $g(1) = 5$, and $\int_0^1 g(x)\,\mathrm{d}x = 2$, find the value of $\int_0^1 xg'(x)\,\mathrm{d}x$.

228. Try each of the following methods to find $\int \sin(x)\cos(x)\,\mathrm{d}x$. (They are all possible.)

(a) Substitue $u = \sin(x)$, so $\mathrm{d}u = \cos(x)\,\mathrm{d}x$ and the integral is $\int u\,\mathrm{d}u$.

(b) Substitue $u = -\cos(x)$, so $\mathrm{d}u = \sin(x)\,\mathrm{d}x$, and the integral is $\int -u\,\mathrm{d}u$.

(c) Substitute $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$, so the integral is $\frac{1}{2}\int \sin(2x)\,\mathrm{d}x$.

(d) Do integration by parts with $u = \sin(x)$ and $\mathrm{d}v = \cos(x)\,\mathrm{d}x$.

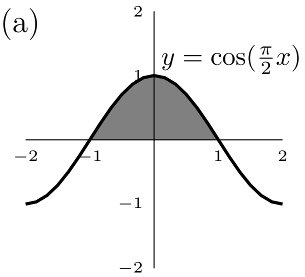
(e) Do integration by parts with $u = \cos(x)$ and $\mathrm{d}v = \sin(x)\,\mathrm{d}x$.

☆(f) Compare your answers to parts (a) - (e).

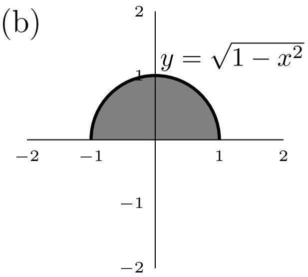
229. Find $\displaystyle\int 4x\cos(2-3x)\,\mathrm{d}x$ and $\displaystyle\int (2-3x)\cos(4x)\,\mathrm{d}x$.

230. Give the area of each of the following shapes:

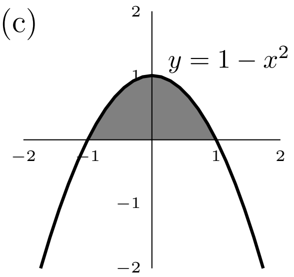
(a)



(b)



(c)



The area between two curves of the form $y = f(x)$ is $\displaystyle\int_{\text{left}}^{\text{right}} \big(\text{top}(x) - \text{bottom}(x)\big)\,\mathrm{d}x$.

The area between two curves of the form $x = g(y)$ is $\displaystyle\int_{\text{bottom}}^{\text{top}} \big(\text{right}(y) - \text{left}(y)\big)\,\mathrm{d}y$.

231. Find the area of the region bounded by $y = e^x, y = x+5, x = -4$, and $x = 0$ (that is, the area between $y = e^x$ and $y = x+5$ with $-4 \le x \le 0$).

232. What is the area of the region bounded by the curves $y = 20 - x^4$ and $y = 4$?

233. Find the area of the region bounded by the curves $x = y^2$ and $x = 1 + y - y^2$.

234. Calculate the area of...

(a) the region bounded by the curves $y = x^2, y = 4x, x = 2, x = 3$.

(b) the region bounded by the curves $y = x^2, y = 4x, y = 1, y = 4$.

(c) the region bounded by the curves $y = x^2$ and $y = 4x$.

The volume of a solid can be calculated as

$$V = \int_{\text{left}}^{\text{right}} \big(\text{cross-section area}\big)\,\mathrm{d}x = \int_{\text{bottom}}^{\text{top}} \big(\text{cross-section area}\big)\,\mathrm{d}y.$$

For a "solid of revolution", the "disk method" uses

$$\pi \cdot (\text{radius})^2$$

as the cross-sectional area.

235. Find the volume of the solid formed by revolving (rotating) the region bounded by $y = 1 - x^2$ and $y = 0$ around the x-axis.

236. Find the volume of the solid formed by revolving the domain

$$\{(x,y) : x \ge 0, 2x \le y \le 6\}$$

around the y-axis.

For Winter 2023, you will not be asked about solids like the ones in Tasks 237 and 238.

☆237. For the solid formed by rotating the region from Task 234(c) around the x-axis,

(a) Set up an integral $\int ... \, dx$ for the volume using the washer method.

(b) and evaluate this integral.

☆238. For the solid formed by rotating the region from Task 234(c) around the y-axis,

(a) set up an integral $\int ... \, dy$ for the volume using the washer method.

(b) and evaluate this integral.

239. Calculate each of the following integrals.
Some* require substitution, some** require parts, and some do not need either.

(a) $\int (x^4 + x^{1/2} + 4 + x^{-1}) \, dx$

(b) $\int \left(x^2 + \sqrt{x} + \frac{\ln(81)}{\ln(3)} + \frac{1}{x}\right) dx$

(c) $\int (t + e^t) \, dt$

(d) $\int (t \cdot e^t) \, dt$

(e) $\int (t^3 + e^{3t}) \, dt$

☆(f) $\int (t^3 \cdot e^{3t}) \, dt$

(g) $\int \frac{x}{x^2+1} \, dx$

(h) $\int \frac{x}{x^2-1} \, dx$

(i) $\int \frac{x^2-1}{x} \, dx$

(j) $\int \frac{1}{x^2-1} \, dx$

☆(k) $\int \frac{1}{x^2+1} \, dx$

(ℓ) $\int \frac{y}{\sqrt{y^2+1}} \, dy$

☆(m) $\int \frac{1}{\sqrt{y^2+1}} \, dy$

(n) $\int t \ln(t) \, dt$

(o) $\int \frac{3t-12}{\sqrt{t^2-8t+6}} \, dt$

(p) $\int \frac{1}{\sqrt{x-1}} \, dx$

(q) $\int \frac{x}{\sqrt{x-1}} \, dx$

(r) $\int y^3 \, dy$

(s) $\int y(y+1)(y-1) \, dy$

(t) $\int x \sin(2x) \, dx$

(u) $\int x^3 \sin(2x^4) \, dx$

☆(v) $\int x^7 \sin(2x^4) \, dx$

☆(w) $\int \sin(2x^4) \, dx$

(x) $\int e^{5x} \cos(e^{5x}) \, dx$

☆(y) $\int x^5 \cos(x) \, dx$

(z) $\int e^{8\ln(t)} \, dt$

* g, h, m, o, p, q, u, x.   ** d, f, ℓ, n, t, v, y.