Math 1688

2 December 2021

Warm-up:
Vector subtraction, plane perpendicular to line, scalar multiples.

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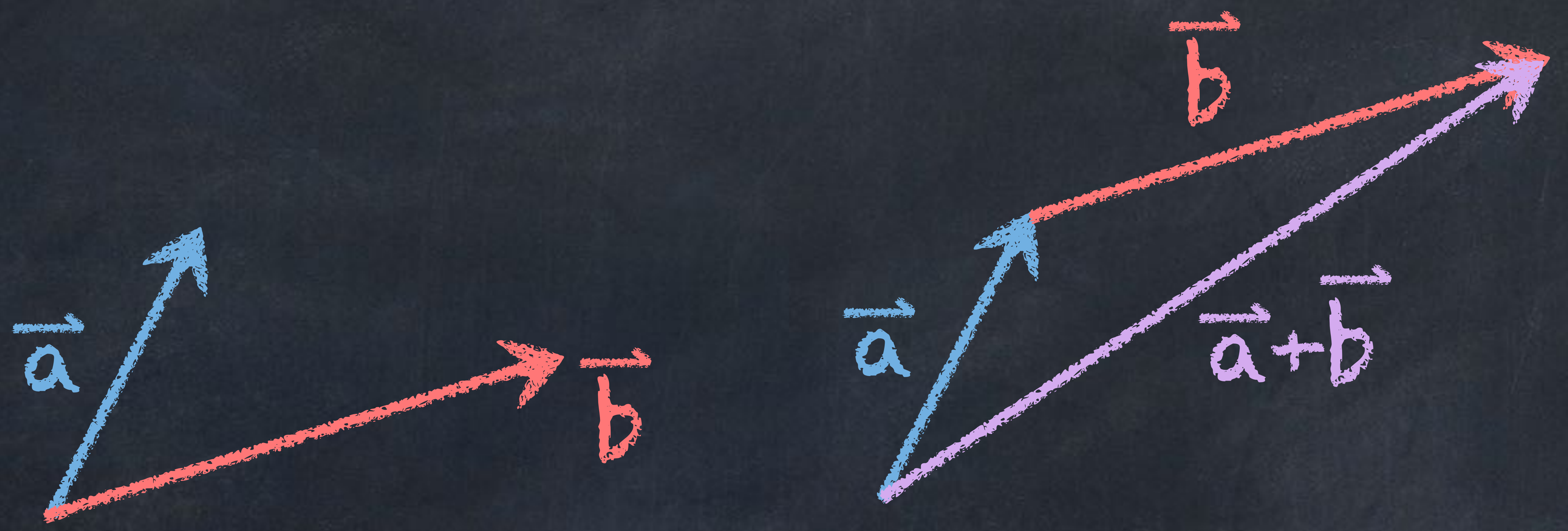
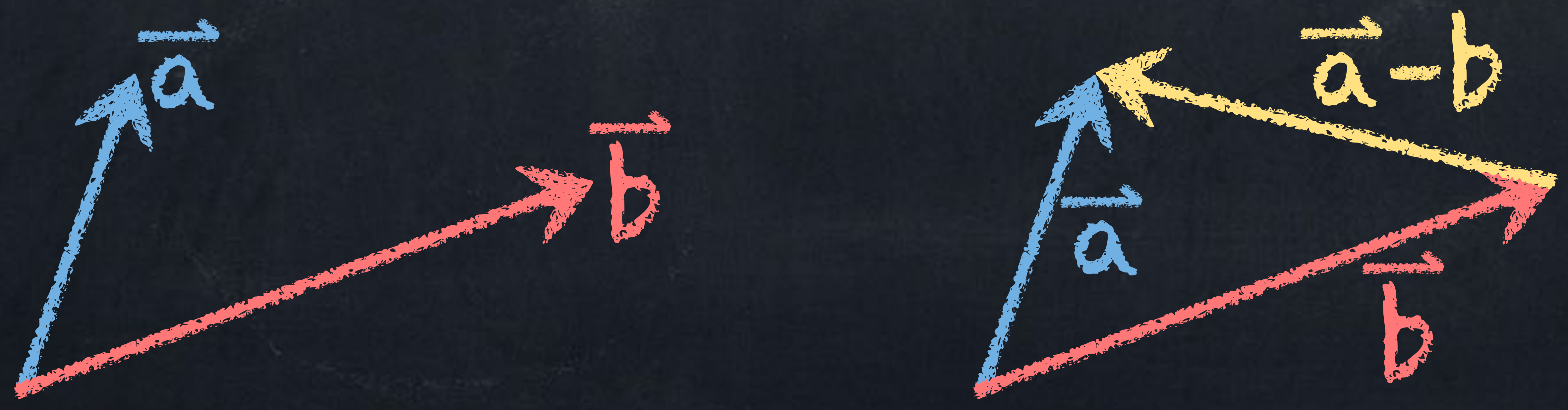
Last two weeks: + and -

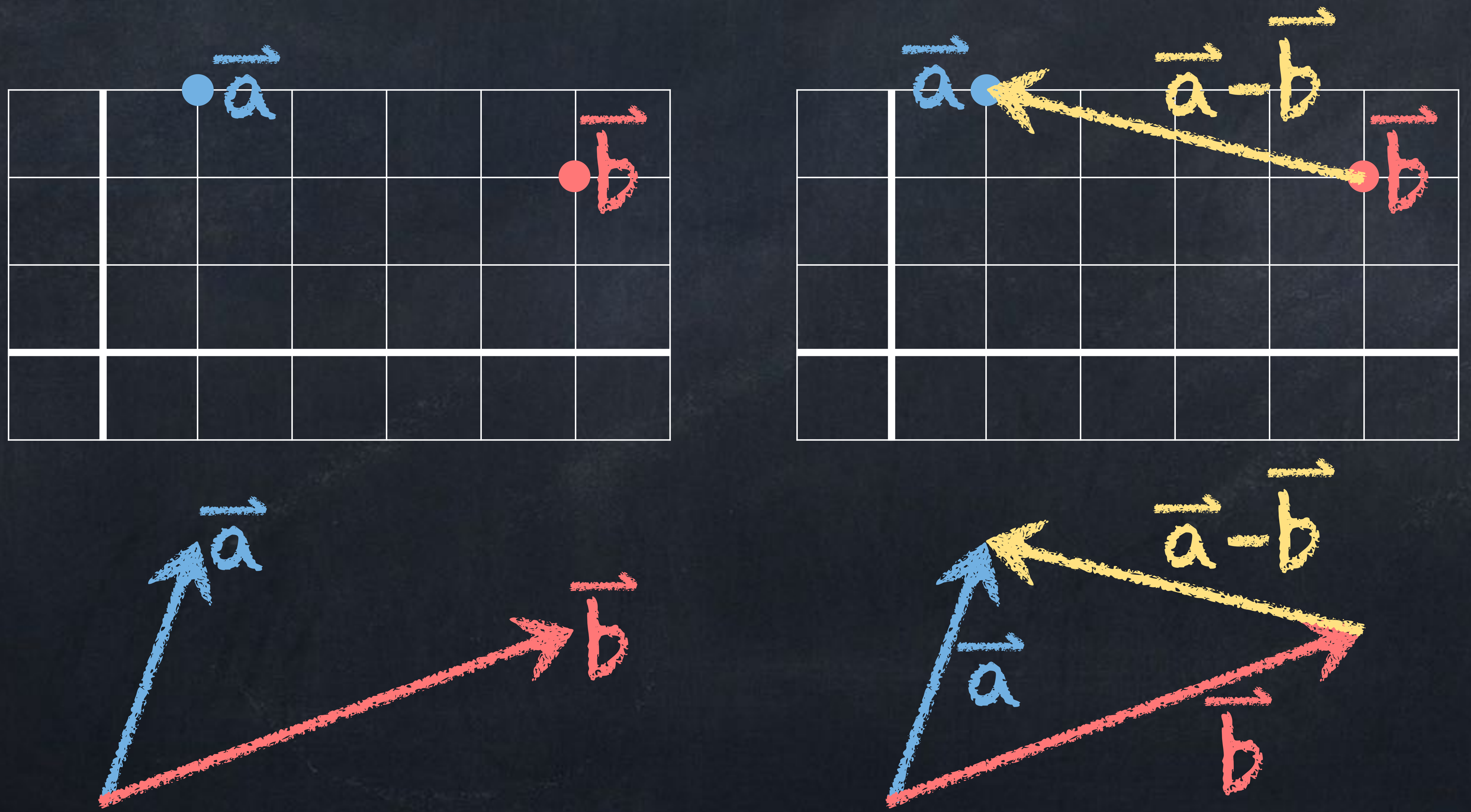
- In terms of arrows, we have "tip-to-tail addition". Example:



- If $\overrightarrow{a}, \overrightarrow{b}$ start at the same point, then $\overrightarrow{a} - \overrightarrow{b}$ points from the end of $\overrightarrow{b}$ to the end of $\overrightarrow{a}$.



If we think of $\overrightarrow{a}$ and $\overrightarrow{b}$ as points, then $\overrightarrow{a} - \overrightarrow{b}$ goes from $\overrightarrow{b}$ to $\overrightarrow{a}$.



Last two weeks: multiplying

We *never* combine $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}4\\5\\6\end{bmatrix}$ into $\begin{bmatrix}(1)(4)\\(2)(5)\\(3)(6)\end{bmatrix} = \begin{bmatrix}4\\10\\18\end{bmatrix}$ in this class.

So far we have talked about

- **scalar multiplication** $7\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}(7)(1)\\(7)(2)\\(7)(3)\end{bmatrix} = \begin{bmatrix}7\\14\\21\end{bmatrix}$

and

- **dot product** $\begin{bmatrix}1\\2\\3\end{bmatrix} \cdot \begin{bmatrix}4\\5\\6\end{bmatrix} = (1)(4) + (2)(5) + (3)(6) = 32$
  - Also, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(\text{angle})$.

Last week: collections/sets

Examples:

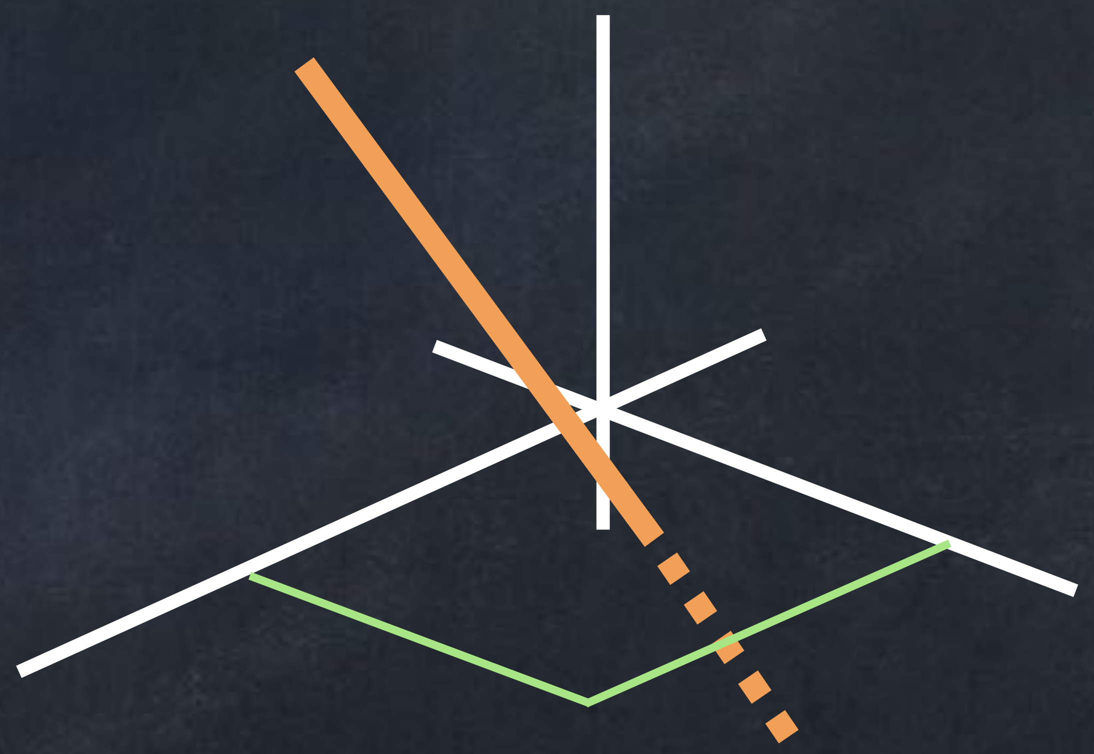
- $A = \left\{ \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} 1 \\ 31 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \end{bmatrix} \right\}$ is a collection of four vectors.
- $B = \{5, 9, 3, 8, 7\}$ is a collection of 5 numbers.
- $C = \{5t : t \in \mathbb{N}\}$ is a set of infinitely many numbers (5, 10, 15, …).
- $D = \left\{ \begin{bmatrix} 4t \\ 10t \end{bmatrix} : t \in \mathbb{R} \right\}$ is a collection of infinitely many vectors that

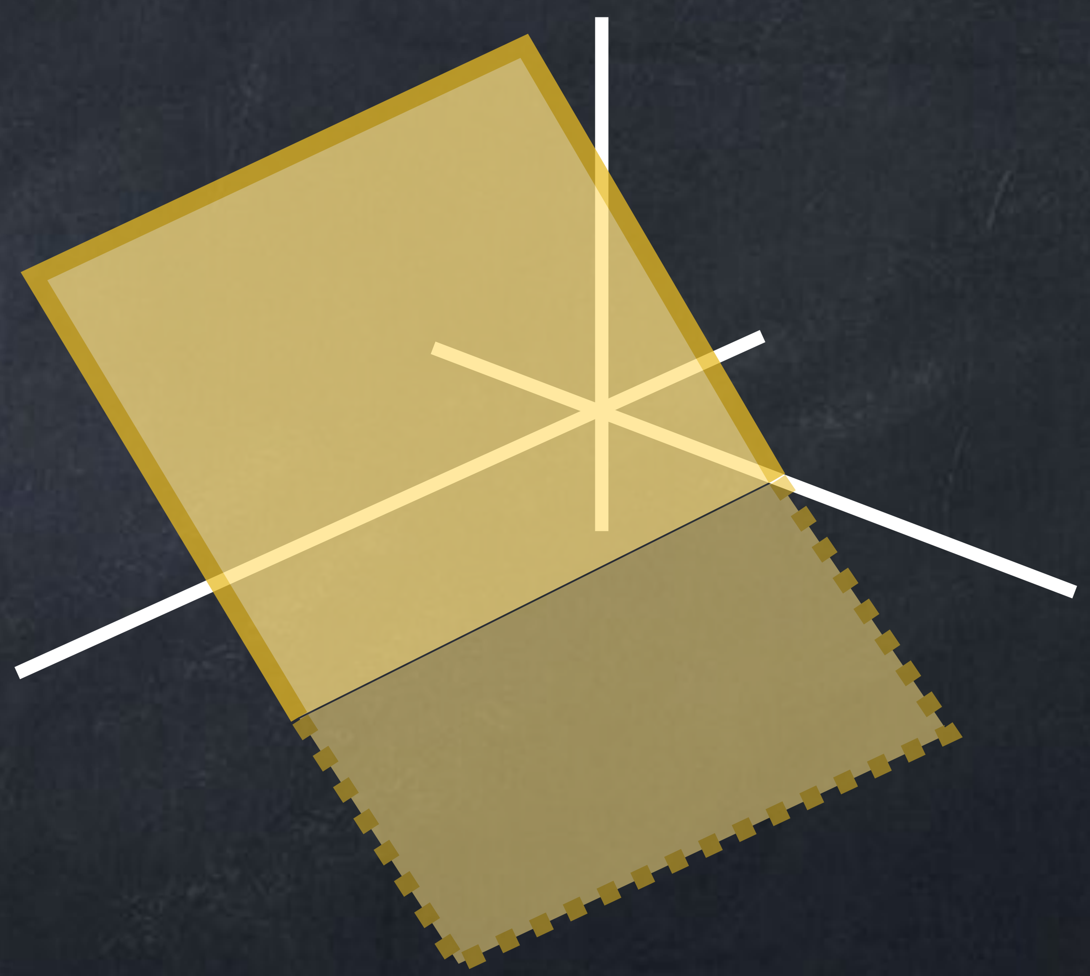
  includes $\begin{bmatrix} 4 \\ 10 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ -10 \end{bmatrix}$ (but not $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$).

Lines and planes

You will need to be able to work *both* visually *and* with equations/symbols about

- lines in 2D
- lines in 3D
- planes in 3D

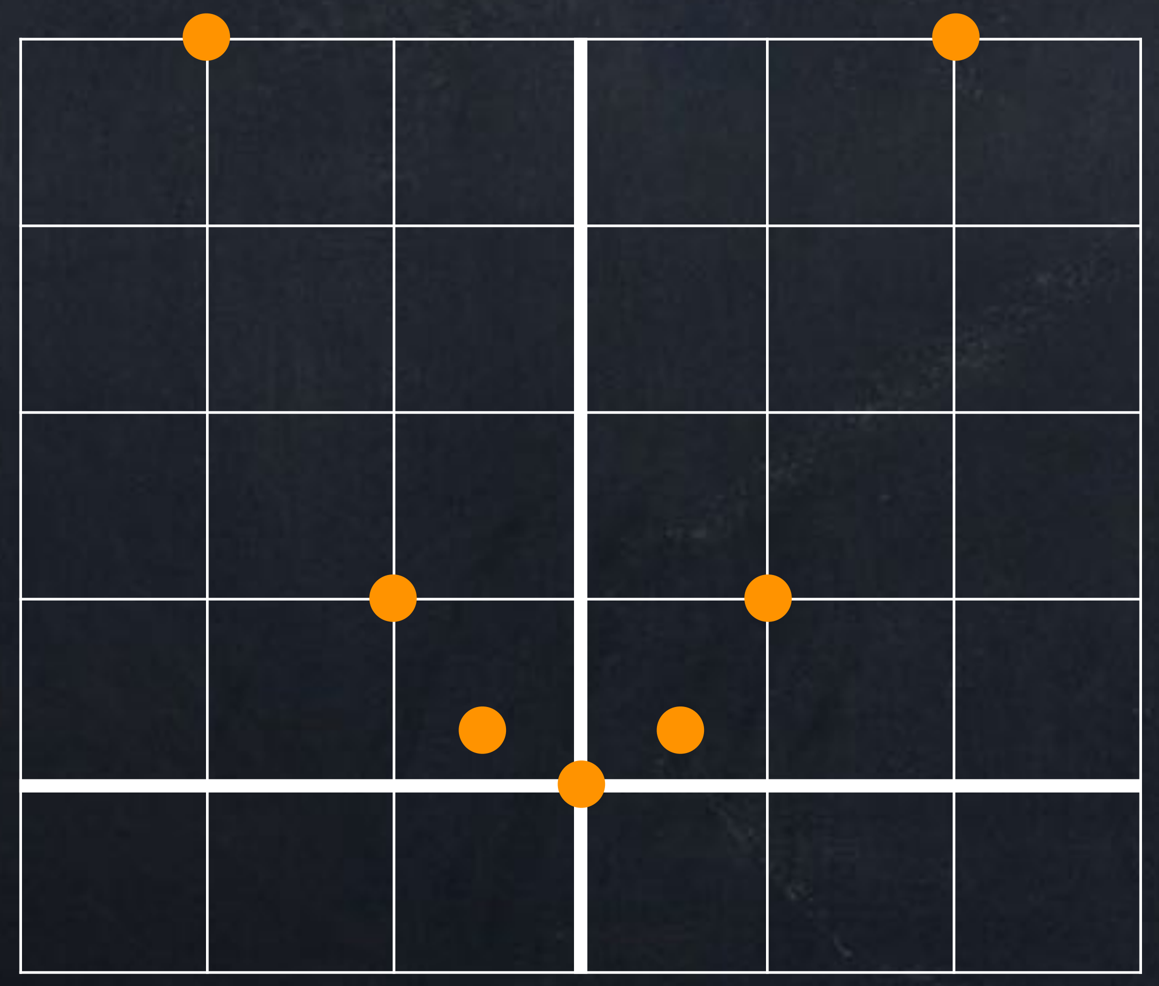


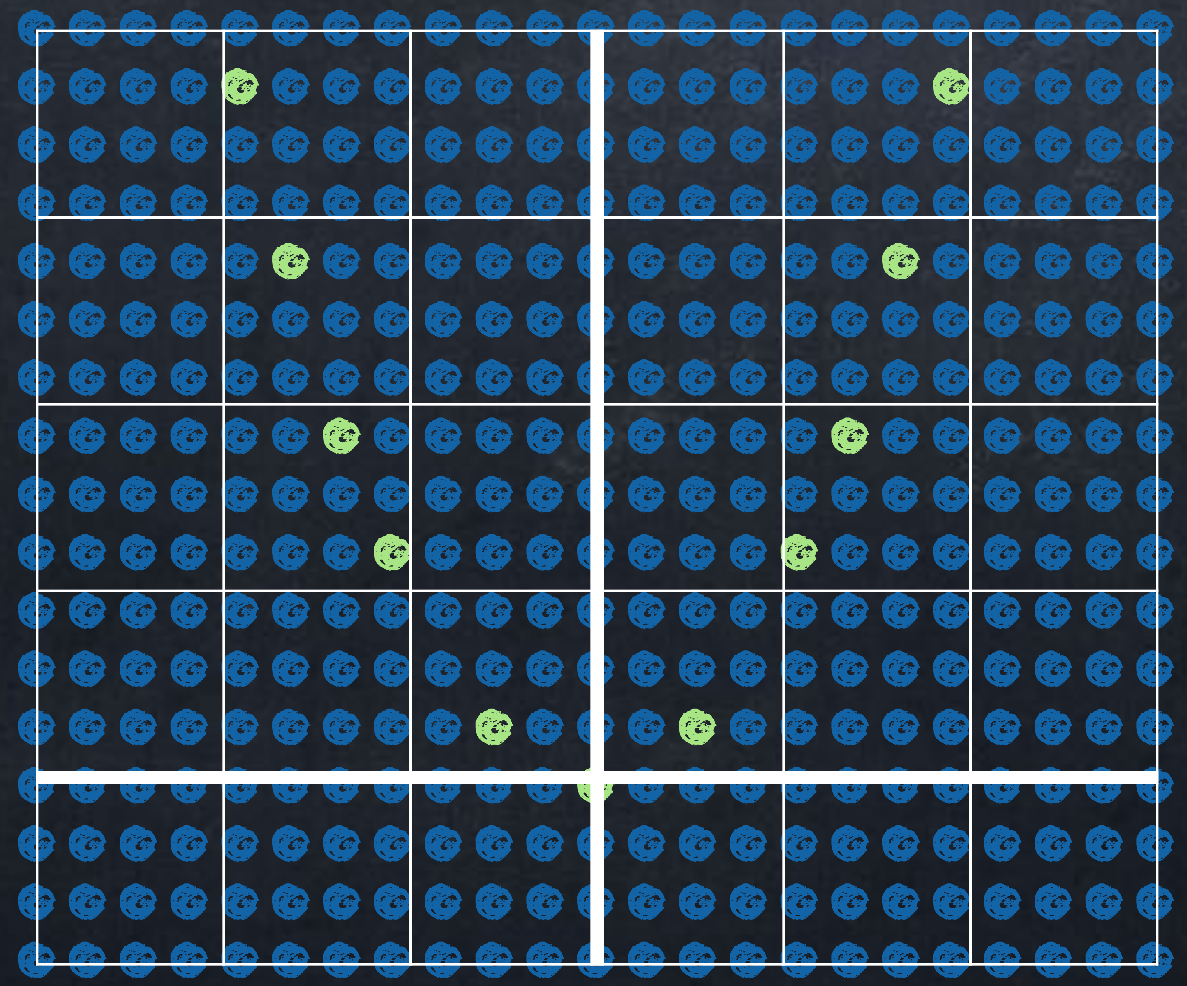


Equations of shapes

There are two ways to think about, for example, $y = x^2$.

- For each specific x-value, you get a y-value, and this tells you one point on the curve.
- For any point (x, y) anywhere, we test whether $y = x^2$ is true for those numbers.





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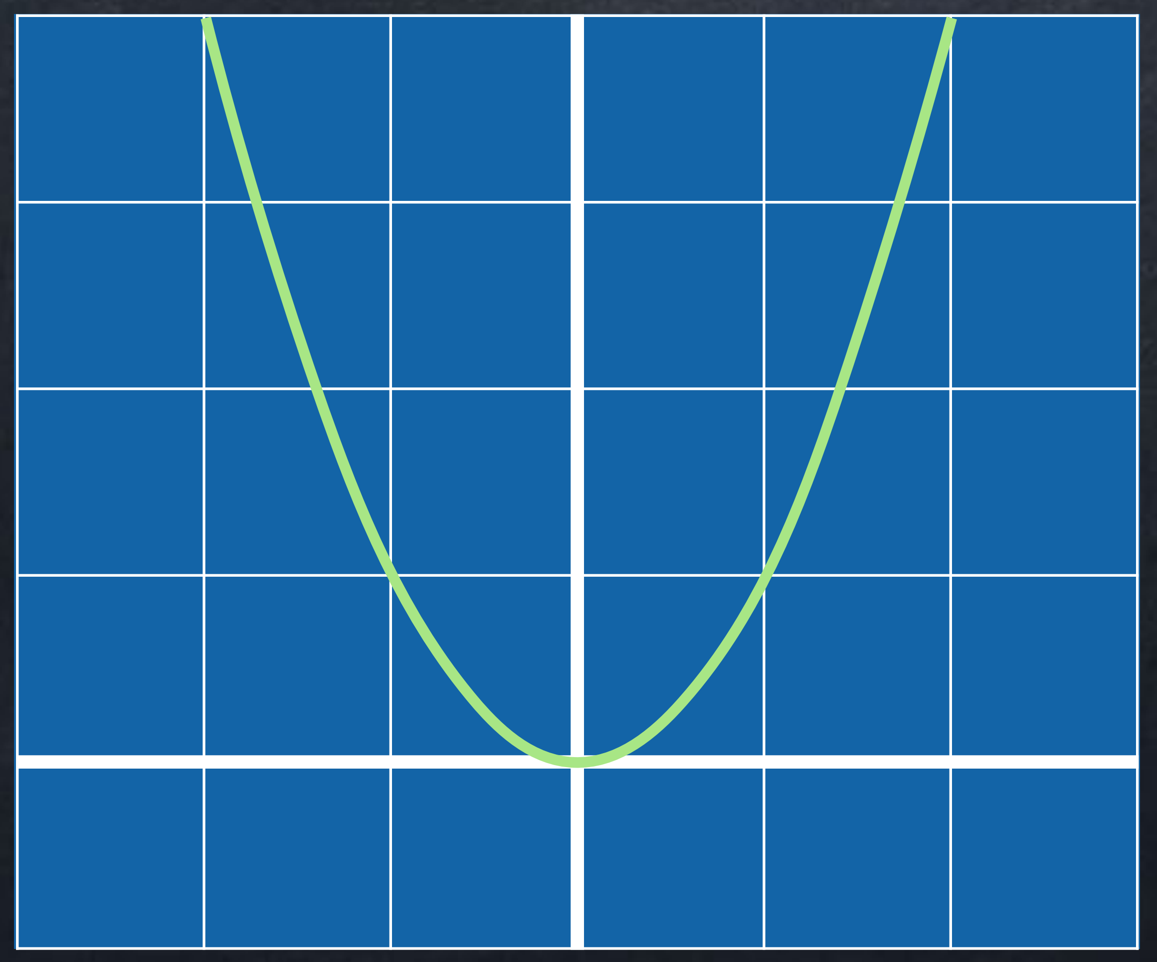

Lines

A **direction vector** for a line is a vector that is parallel (or anti-parallel) to the line.

- The length of this vector does not matter: if $\overrightarrow{v}$ is a direction vector for a line, then $2\overrightarrow{v}$ and $-0.3\overrightarrow{v}$ and are also directions vectors for that line.

- If we only know
  - a direction vector for a line L

  this is not enough to know what the line L is—there will be many lines that all have that same direction vector (these lines will be parallel).

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- If we only know
  - a point on a line L

  this is not enough to know what the line L is—there will be many lines that go through that point.

Lines

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- If we know
  - a point on a line L

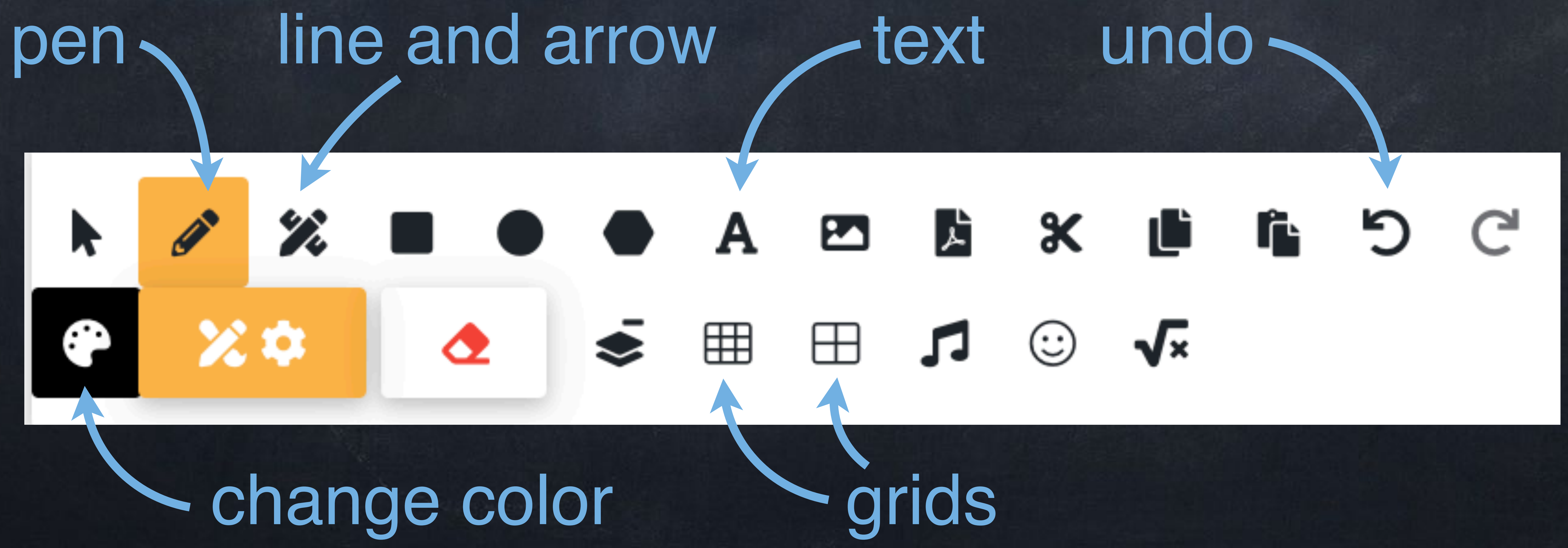
  and
  - a direction vector for a line L

  then there is exactly one line L that fits both of these.

Whiteboard website

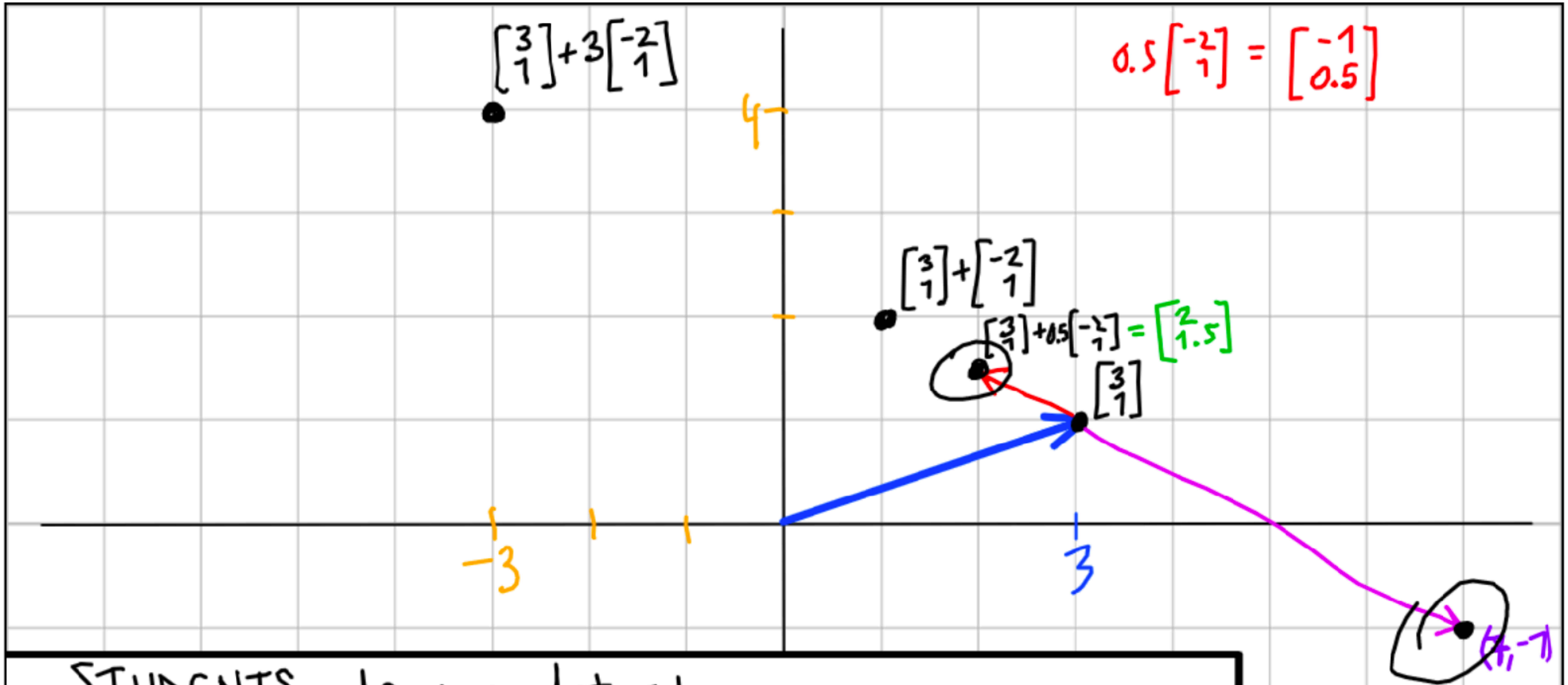
http://whiteboard.fi/math1688

- If you are asked for a room code, it is **math1688**
- Type your first and last name, then click Join Whiteboard Class
- Click TOGGLE TEACHER WHITEBOARD



Draw the arrow [3,1] starting at the origin.

Draw the arrow [-2,1] starting at (3,1).

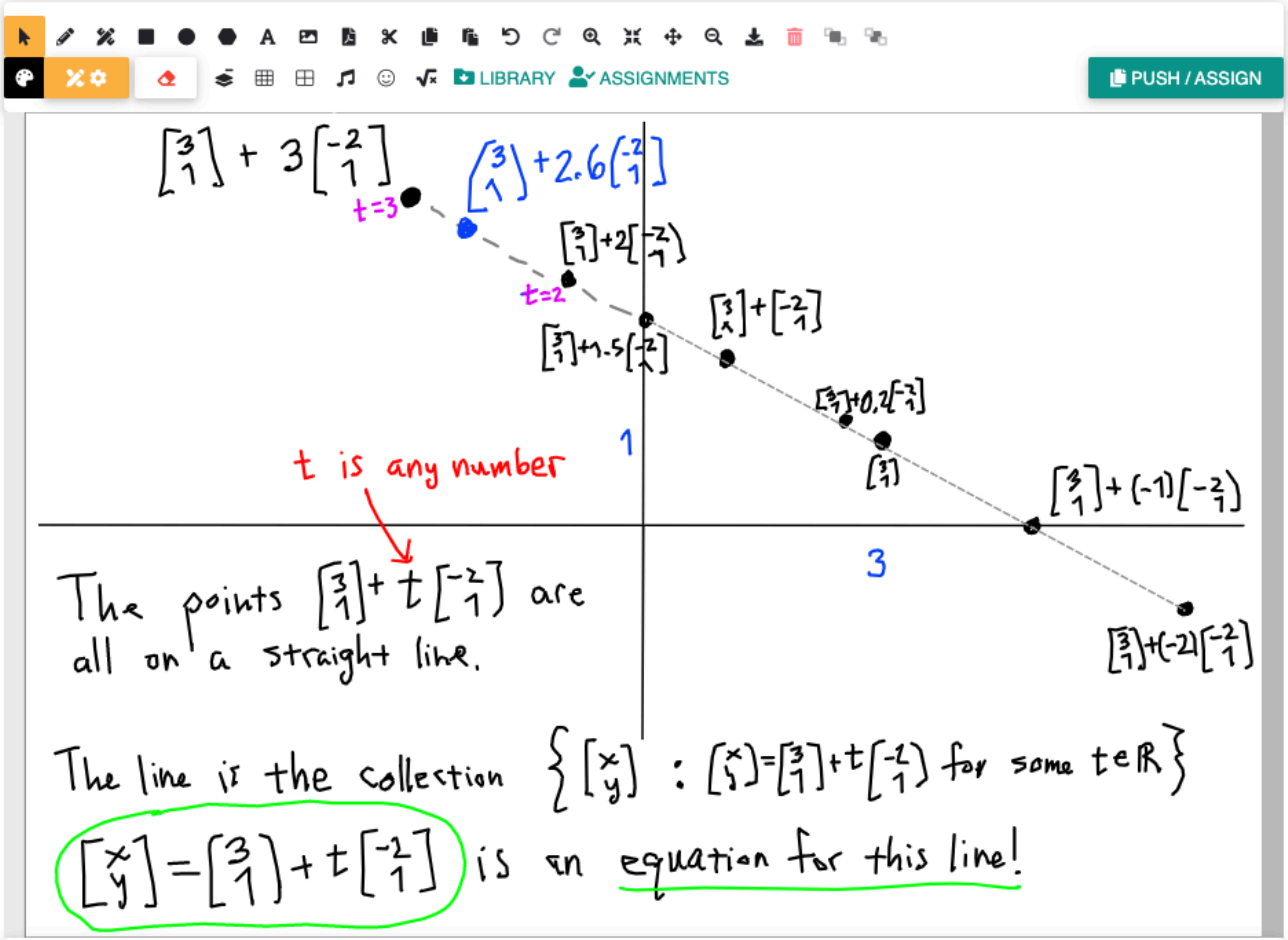


STUDENTS, draw a dot at

$$\begin{bmatrix}3\\1\end{bmatrix}+0.5\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}+\begin{bmatrix}-1\\0.5\end{bmatrix}=\begin{bmatrix}3-1\\1+0.5\end{bmatrix}=\begin{bmatrix}2\\1.5\end{bmatrix}$$

and at

$$\begin{bmatrix}3\\1\end{bmatrix}+(-2)\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}+\begin{bmatrix}-2\cdot-2\\-2\times1\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}+\begin{bmatrix}4\\-2\end{bmatrix}=\begin{bmatrix}7\\-1\end{bmatrix}$$


LIBRARY
ASSIGNMENTS
PUSH / ASSIGN
$\begin{bmatrix}3\\1\end{bmatrix} + 3\begin{bmatrix}-2\\1\end{bmatrix}$
t=3
$\begin{bmatrix}3\\1\end{bmatrix} + 2.6\begin{bmatrix}-2\\1\end{bmatrix}$
$\begin{bmatrix}3\\1\end{bmatrix} + 2\begin{bmatrix}-2\\1\end{bmatrix}$
t=2
$\begin{bmatrix}3\\1\end{bmatrix} + 1.5\begin{bmatrix}-2\\1\end{bmatrix}$
$\begin{bmatrix}3\\1\end{bmatrix} + \begin{bmatrix}-2\\1\end{bmatrix}$
$\begin{bmatrix}3\\1\end{bmatrix} + 0.2\begin{bmatrix}-2\\1\end{bmatrix}$
$\begin{bmatrix}3\\1\end{bmatrix}$
1
3
$\begin{bmatrix}3\\1\end{bmatrix} + (-1)\begin{bmatrix}-2\\1\end{bmatrix}$
$\begin{bmatrix}3\\1\end{bmatrix} + (-2)\begin{bmatrix}-2\\1\end{bmatrix}$
t is any number
The points $\begin{bmatrix}3\\1\end{bmatrix} + t\begin{bmatrix}-2\\1\end{bmatrix}$ are all on a straight line.
The line is the collection $\left\{ \begin{bmatrix}x\\y\end{bmatrix} : \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix} + t\begin{bmatrix}-2\\1\end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$
$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix} + t\begin{bmatrix}-2\\1\end{bmatrix}$ is an equation for this line!

Named vectors

We have talked about some special vectors in 2D and 3D:

- $\vec{i} = [1, 0]$
- $\vec{j} = [0, 1]$

- $\vec{i} = [1, 0, 0]$
- $\vec{j} = [0, 1, 0]$
- $\vec{k} = [0, 0, 1]$.

There is one other vector letter that has a special meaning:

$$\vec{r} = \begin{bmatrix} x \\ y \end{bmatrix} \text{ in 2D, } \quad \text{and} \quad \vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ in 3D.}$$

This allows us to write very short equations.

Lines

In 2D the line through $\overrightarrow{p} = [x_0, y_0]$ parallel to $\overrightarrow{d} = [a, b]$ has equation

- $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t \begin{bmatrix} a \\ b \end{bmatrix}$
- or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 + ta \\ y_0 + tb \end{bmatrix}$
- or $x = x_0 + ta, y = y_0 + tb$
- or $\vec{r} = \overrightarrow{p} + t\,\overrightarrow{d}$.

The line through the point $\overrightarrow{p}$ with direction vector $\overrightarrow{d}$ has equation

$$\vec{r} = \overrightarrow{p} + t\,\overrightarrow{d}$$

in 2D or 3D!

Give an equation for the line through $(5, -2)$ with direction vector $[4,7]$.

$$\vec{r} = \begin{bmatrix} 5 \\ -2 \end{bmatrix} + t \begin{bmatrix} 4 \\ 7 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix} + t \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5+4t \\ -2+7t \end{bmatrix} \quad \text{or} \quad x = 5+4t \text{ and } y = -2+7t$$

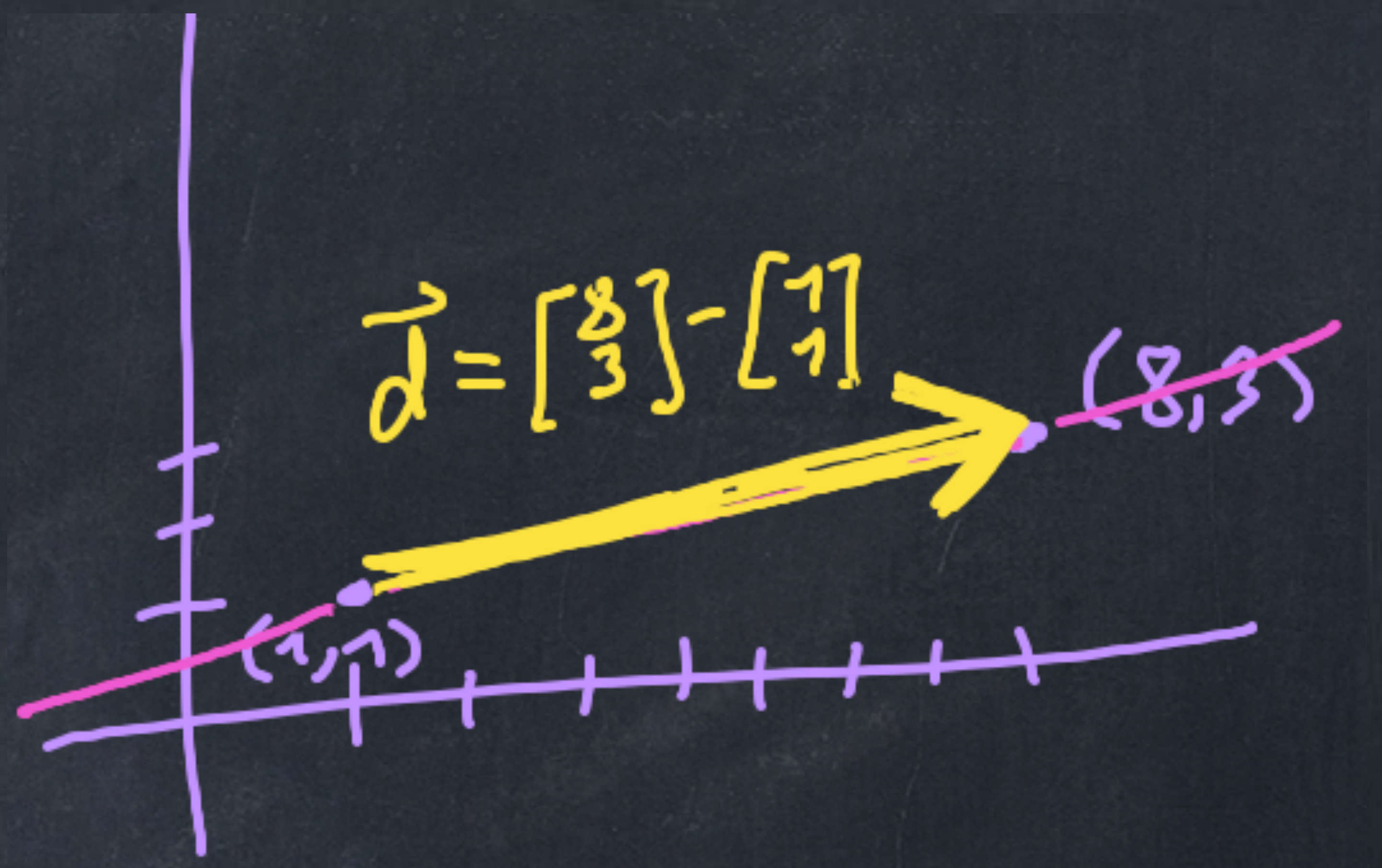
Since $t = (x-5)/4$ and $t = (y+2)/7$, this line is also

$$\frac{x-5}{4} = \frac{y+2}{7} \quad \text{or} \quad y = \frac{7}{4}x - \frac{43}{4}.$$

Give a vector equation for the line through $(1,1)$ and $(8,3)$.

We want a point on this line and a direction vector for this line.

Point: (1,1)



Dir. vec.: $\begin{bmatrix}8\\3\end{bmatrix} - \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}7\\2\end{bmatrix}$ is the arrow from (1,1) to (8,3).

Vector equation for the line:

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} + t\begin{bmatrix}7\\2\end{bmatrix} \quad \text{or} \quad \vec{r} = \begin{bmatrix}1\\1\end{bmatrix} + t\begin{bmatrix}7\\2\end{bmatrix}.$$