Linear Algebra, Winter 2021

List 1

Algebra review, trig values, complex numbers.

1. Classify each of the following as an "expression", "equation", or "inequality".

   (a) $5x^2 + 2$
   (b) $8 = 9$
   (c) $3x^5 - \sqrt{x}$
   (d) $\sin(\pi)$
   (e) $9x^3 - 5 + i \leq 0$
   (f) $9x^3 - 5 + i > 0$
   (g) $9x^3 - 5 + i = 0$
   (h) $4^x = 2x - 17$

2. Which of the following are true for **all** real values of the variables?

   (a) $2x = x + x$
   (b) $2(x + y) = 2x + y$
   (c) $(x - y)^2 = x^2 - 2xy + y^2$
   (d) $(6 + a)/2 = 3 + a/2$
   (e) $-(y + 2) = -y + 2$
   (f) $-(a + b)^2 = (-a + b)^2$
   (g) $x^3 + 3x = x + x$
   (h) $k^{-2} = 1/k^2$
   (i) $x^{a+2} = x^a \times x^2$

3. Compute the following values:

   (a) $\cos(0)$
   (b) $\sin(0)$
   (c) $\cos(30°)$
   (d) $\cos(45°)$
   (e) $\cos(60°)$
   (f) $\cos(\pi/3)$
   (g) $\cos(\pi/2)$
   (h) $\sin(\pi/2)$
   (i) $\sin(120°)$
   (j) $\sin(5\pi/6)$
   (k) $\sin(180°)$
   (ℓ) $\sin(4\pi/3)$
   (m) $\cos(315°)$
   (n) $\cos(-45°)$
   (o) $\cos(675°)$
   (p) $\arccos(\frac{\sqrt{3}}{2})$

The number "i" satisfies $i \times i = -1$. If a and b are any real numbers (including zero), then the number "$a + bi$" is called a **complex number**. The **real part** of this number is a, the **imaginary part** of this number is b, and the **conjugate** of this number is $a - bi$.

For a complex number z, we write $\mathrm{Re}(z)$ for its real part, $\mathrm{Im}(z)$ for its imaginary part, and $\overline{z}$ (spoken as "z bar") for its conjugate.

4. Write the following in rectangular form $a + bi$ (also called Cartesian form):

   (a) $(-6 + 5i) + (2 - 4i)$
   (b) $(1 + 2i)(2 + 3i)$
   (c) $(-5 + 2i) - (2 - i)$
   (d) $(2 - 3i)(2 + 3i)$
   (e) $(1 + i)(2 - i)(3 + 2i)$
   (f) $(1 - 2i)^3$
   (g) $(-2i)^6$
   (h) $\overline{5 + 6i}$
   (i) $\overline{-1 - 9i}$
   (j) $\overline{3i}$
   (k) $\overline{12}$
   (ℓ) $(2 - 3i)(\overline{2 - 3i})$
   (m) $\mathrm{Re}(2i - 7)$
   (n) $\mathrm{Im}\big((3 + 2i)(5i)\big)$
   (o) $\mathrm{Re}(i^2)$

5. Write $\dfrac{1 + 2i}{2 - 3i}$ in the form $a + bi$.

The **modulus** of z, written $|z|$, is the distance between z and 0 on the complex plane (that is, between $(0,0)$ and (a,b) on an xy-plane). The **argument** of z, written $\arg(z)$, is the angle between the positive real axis and the line from 0 to z.

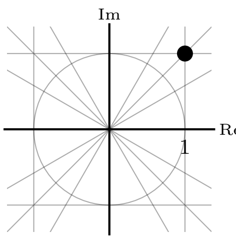
We write $r \cdot e^{(\theta \cdot i)}$ for the complex number $r\cos(\theta) + r\sin(\theta)\, i$.
Notice that $re^{\theta i}$ has modulus r and argument θ.

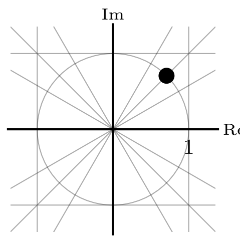
6. Compute $|1 - \sqrt{3}i|$ and $\arg(1 - \sqrt{3}i)$.

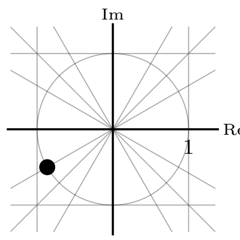
7. Write the following numbers in polar form $re^{i\theta}$ (also called trigonometric form):
   (a) $-3i$
   (b) $1 + \sqrt{3}i$
   (c) $2 - 2\sqrt{3}i$
   (d) $\dfrac{\sqrt{3} - i}{7}$
   (e) $\cos(\frac{5}{11}\pi) + i\sin(\frac{5}{11}\pi)$
   (f) $\sqrt{-1}$

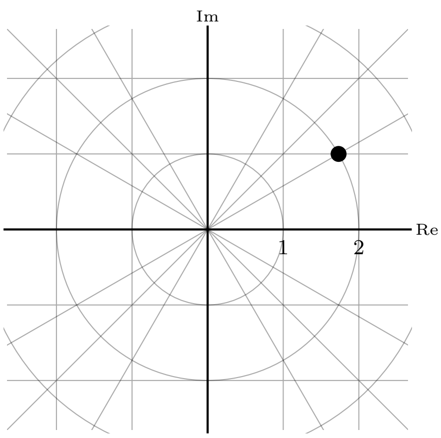
8. Write the following in rectangular form:
   (a) $e^{\frac{\pi}{4}i}$
   (b) $2e^{i\pi/6}$
   (c) $5e^{-i\pi/3}$
   (d) $-8e^{\pi i}$
   (e) $\sqrt{9} + \sqrt{-9}$

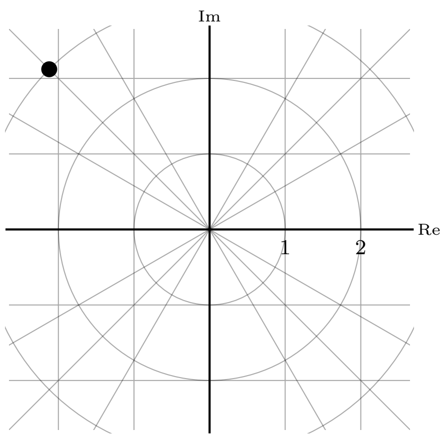
9. Write each number in both rectangular and polar form:

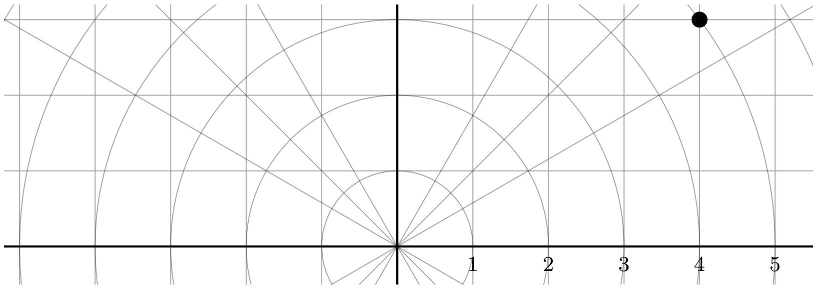
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

10. On a complex plane, draw the number(s)...
    (a) $\dfrac{1+i}{\sqrt{2}}$ and $\dfrac{-1-i}{\sqrt{2}}$
    (b) $\dfrac{1-i}{\sqrt{2}}$ and $\dfrac{-1+i}{\sqrt{2}}$
    (c) $3-i$
    (d) $\sqrt{2}-\sqrt{2}i$ and $-\sqrt{2}+\sqrt{2}i$
    (e) $-\dfrac{19}{29}-\dfrac{25}{29}i$
    (f) $3-i$ and $3+i$
    (g) $\dfrac{1-25i}{6}$
    (h) $i-\sqrt{2}$ and $i+\sqrt{2}$

11. Compute the modulus (also called Euclidean norm) of the number...
    (a) $2+7i$
    (b) $\dfrac{4+i}{3+2i}$
    (c) $(1+\sqrt{2}i)$
    (d) $\dfrac{(3-i\sqrt{3})^2}{(\sqrt{2}+2i)^3}$