Linear Algebra, Winter 2021

List 2

Complex numbers, intro to polynomials.

12. For $z = \frac{\sqrt{7}}{2} + \frac{\sqrt{11}}{3}i$, calculate $z + \overline{z}$. $\boxed{\sqrt{7}}$

13. For $z = 9e^{(\pi/8)i}$, calculate $z \cdot \overline{z}$. $\boxed{81}$

14. For $z = 6e^{(\pi/52)i}$ and $w = 3e^{(-\pi/52)i}$, calculate zw. Give your answer in rectangular form. $\boxed{18}$

15. For $z = 1 + i$ and $w = e^{(\pi/4)i}$, calculate

    (a) $\arg(z)$ $\boxed{\pi/4}$

    (b) $\arg(w)$ $\boxed{\pi/4}$

    (c) $\arg(zw)$ $\boxed{\pi/2}$

    (d) $z + w$ $\boxed{\left(1 + \frac{1}{\sqrt{2}}\right) + \left(1 + \frac{1}{\sqrt{2}}\right)i}$ or $\boxed{(1 + \sqrt{2})e^{(\pi/4)i}}$

    (e) $|w|$ $\boxed{1}$

    (f) $|z - w|$ $\boxed{\sqrt{2} - 1}$

    (g) $|zw|$ $\boxed{\sqrt{2}}$

    (h) $|z/w|$ $\boxed{\sqrt{2}}$

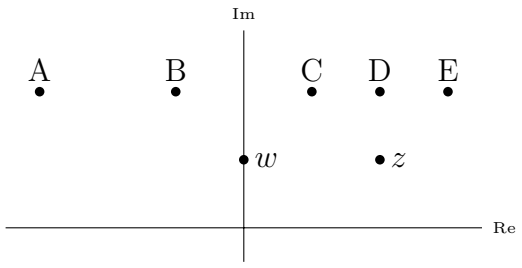
    (i) zw $\boxed{\sqrt{2}i}$

    (j) z/w $\boxed{\sqrt{2}}$

    (k) $z\,\overline{w}$ $\boxed{\sqrt{2}}$

    (ℓ) $\overline{z}\,w$ $\boxed{\sqrt{2}}$

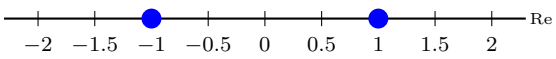
16. Which of the points A - E below could be $z + w$? Which could be zw?



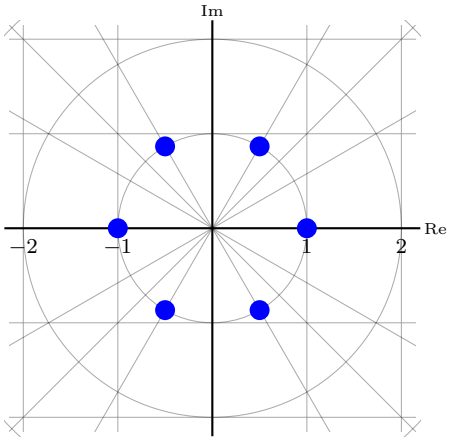
    $\boxed{\text{D}}$ is $z + w$. $\boxed{\text{B}}$ is the only option that could be zw.

17. Write $(1 + i)^{11}$ $\boxed{-32 + 32i}$ and $\left(\frac{\sqrt{3} - i}{1 + i}\right)^6$ $\boxed{-2 + 2i}$ in rectangular form. (Hint: use de Moivre's formula.)

18. (a) On a real number line (like the blank one shown below), put a dot at every point x for which $x^6 = 1$.



(b) On a complex plane (like the blank one shown below), put a dot at every point z for which $z^6 = 1$.



19. A cannonball fired at 400 m/s at an angle of 52° will have an initial vertical velocity of $400\sin(52°) \approx 315.2$ m/s, and it will have a height of

$$h(t) = \frac{-9.8}{2}t^2 + 315.2\,t$$

meters after t seconds. How many seconds will it take for the cannonball to reach the ground?

Without a calculator, $\boxed{\dfrac{2 \times 315.2}{9.8}}$ is good enough. With calculator, $\boxed{64.3265}$.

20. If the width of a rectangle is 5 m more than its length, and the rectangle's area is 84 m^2, what are the length and width of the rectangle?

The solutions to $x(x+5) = 84$ are $x = 7$ and $x = -12$, but length cannot be negative. The $\boxed{\text{length is } 7}$ and so the $\boxed{\text{width is } 12}$.

21. The product of two positive consecutive[1] integers is 380. Find the two numbers.

The solutions to $x(x+1) = 380$ are $x = 19$ and $x = -20$, but the numbers must be positive, so they are $\boxed{19 \text{ and } 20}$.

A **polynomial** in the variable x is a function that can be written in the form

$$\_\_x^n + \_\_x^{n-1} + \cdots + \_\_x^2 + \_\_x + \_\_,$$

where each blank—called a **coefficient**—is a real or complex number (possibly including 0). The **degree** of a polynomial in x is the highest power of x that has a non-zero coefficient.

[1] For example, the numbers 107 and 108 are consecutive.

22. For each of the following, give the degree if the expression is a polynomial in x, and otherwise write "not a polynomial".

(a) $\frac{5}{2}x^3 - 7x + 8$ degree 3

(b) $9x^{10}$ degree 10

(c) $6x^5 + \frac{1}{3}x + 5x^{-2}$ not a polynomial

(d) $3x^2 + \sin(x)$ not a polynomial

(e) $(x^2 + 2x - 1)^3$ degree 6

(f) $5x$ degree 1

(g) 5 degree 0

(h) $\dfrac{8x+1}{2x}$ not a polynomial

(i) $\dfrac{x^3 + 7x}{2}$ degree 3