Linear Algebra, Winter 2021

## List 2

Complex numbers, intro to polynomials.
12. For $z=\frac{\sqrt{7}}{2}+\frac{\sqrt{11}}{3} i$, calculate $z+\bar{z}$.
13. For $z=9 e^{(\pi / 8) i}$, calculate $z \cdot \bar{z}$.
14. For $z=6 e^{(\pi / 52) i}$ and $w=3 e^{(-\pi / 52) i}$, calculate $z w$. Give your answer in rectangular form.
15. For $z=1+i$ and $w=e^{(\pi / 4) i}$, calculate
(a) $\arg (z)$
(e) $|w|$
(i) $z w$
(b) $\arg (w)$
(f) $|z-w|$
(j) $z / w$
(c) $\arg (z w)$
(g) $|z w|$
( k$) z \bar{w}$
(d) $z+w$
(h) $|z / w|$
( $\ell$ ) $\bar{z} w$
16. Which of the points A - E below could be $z+w$ ? Which could be $z w$ ?

17. Write $(1+i)^{11}$ and $\left(\frac{\sqrt{3}-i}{1+i}\right)^{6}$ in rectangular form. (Hint: use de Moivre's formula.)
18. (a) On a real number line (like the blank one shown below), put a dot at every point $x$ for which $x^{6}=1$.

(b) On a complex plane (like the blank one shown below), put a dot at every point $z$ for which $z^{6}=1$.

19. A cannonball fired at $400 \mathrm{~m} / \mathrm{s}$ at an angle of $52^{\circ}$ will have an initial vertical velocity of $400 \sin \left(52^{\circ}\right) \approx 315.2 \mathrm{~m} / \mathrm{s}$, and it will have a height of

$$
h(t)=\frac{-9.8}{2} t^{2}+315.2 t
$$

meters after $t$ seconds. How many seconds will it take for the cannonball to reach the ground?
20. If the width of a rectangle is 5 m more than its length, and the rectangle's area is $84 \mathrm{~m}^{2}$, what are the length and width of the rectangle?
21. The product of two positive consecutive ${ }^{1}$ integers is 380 . Find the two numbers.

A polynomial in the variable $x$ is a function that can be written in the form

$$
\ldots x^{n}+\ldots x^{n-1}+\cdots+\ldots x^{2}+\ldots x+\ldots,
$$

where each blank - called a coefficient - is a real or complex number (possibly including 0 ). The degree of a polynomial in $x$ is the highest power of $x$ that has a non-zero coefficient.
22. For each of the following, give the degree if the expression is a polynomial in $x$, and otherwise write "not a polynomial".
(a) $\frac{5}{2} x^{3}-7 x+8$
(f) $5 x$
(b) $9 x^{10}$
(g) 5
(c) $6 x^{5}+\frac{1}{3} x+5 x^{-2}$
(h) $\frac{8 x+1}{2 x}$
(d) $3 x^{2}+\sin (x)$
(e) $\left(x^{2}+2 x-1\right)^{3}$
(i) $\frac{x^{3}+7 x}{2}$

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[^0]:    ${ }^{1}$ For example, the numbers 107 and 108 are consecutive.

