Linear Algebra, Winter 2021
List 4
*Extra practice: complex numbers and polynomials.*

42. (a) Simplify $x^9 \cdot x^4$.

    (b) Simplify $e^9 \cdot e^4$.

    (c) Simplify $5e^{90} \cdot 7e^{40}$.

    (d) Simplify $5e^{9i} \cdot 7e^{4i}$.

    (e) Simplify $5e^{(\pi/6)i} \cdot 7e^{(\pi/2)i}$.

43. [fast] With $z = 2\sqrt{3}e^{(\pi/3)i} = \sqrt{3} + 3i$ and $w = 2e^{(\pi/6)i} = \sqrt{3} + i$, compute $\dfrac{z}{w}$, giving your answer in either polar form or rectangular form (your choice).

44. [fast] With $z = 2\sqrt{3}e^{(\pi/3)i} = \sqrt{3} + 3i$ and $w = 2e^{(\pi/6)i} = \sqrt{3} + i$, compute $z - w$, giving your answer in either polar form or rectangular form (your choice).

45. Convert the following numbers to polar form, that is, $\_\_ e^{(\_\_ i)}$ where the first blank is a positive number.

    (a) $4\cos(21°) + 4\sin(21°)i$

    (b) $9\cos(-3°) + 9\sin(-3°)i$

    (c) $\cos(\pi/4) + \sin(\pi/4)i$

    (d) $\sqrt{2}\cos(\pi/4) + \sqrt{2}\sin(\pi/4)i$

    (e) $1 + i$

    (f) $4i$

    (g) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

    (h) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

    (i) $1 - \sqrt{3}i$

    (j) $\sqrt{3} - 3i$

    (k) $\overline{-5 - 5i}$

    (ℓ) $\overline{6e^{5\pi/6}}$

    (m) $\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^{10}$

46. Give the real polynomial of the form $\_\_ x^3 + \_\_ x^2 + \_\_ x + \_\_$ whose graph is shown below.
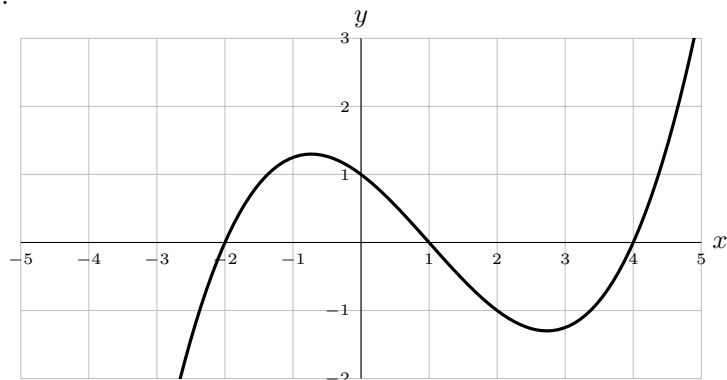
47. Give the real polynomial of the form $x^3 + \_\_ x^2 + \_\_ x + \_\_$ whose roots include the two complex numbers shown below.
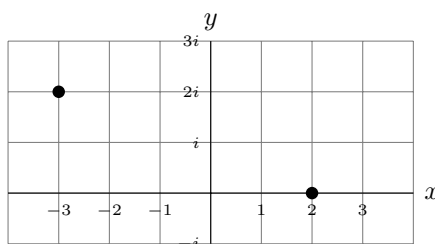
48. Which of the points A - E below could be $z \cdot w$?
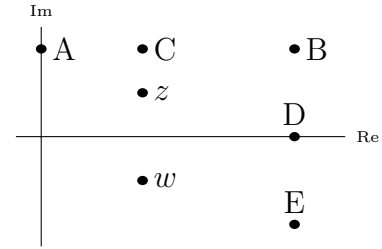
49. [fast] Find the real root(s) of $x^8 + x^6$ and the multiplicity of each root.

50. [medium fast] Find the complex roots of $x^8 + x^6$ and their multiplicities.

51. [slow] One of the roots of $2x^5 - 5x^4 + 10x^2 - 10x + 3$ is 1; what is its multiplicity?

52. (a) [medium fast] Give the quotient when $x^3 + 2x^2 - 7x + 1$ is divided by $x - 2$.

    (b) [fast] Give the remainder when $x^3 + 2x^2 - 7x + 1$ is divided by $x - 2$.

    (c) [slow] Give the remainder when $x^3 + 2x^2 - 7x + 1$ is divided by $x^2 - 2$.

53. [fast] The partial fraction decomposition of $\frac{x^5 - 23x^2 + 15x + 81}{x^2 - 4x + 4}$ is

$$\frac{x^5 - 23x^2 + 15x + 81}{x^2 - 4x + 4} = (x^3 + 4x^2 + 12x + 9) + \frac{3}{x - 2} + \frac{51}{(x - 2)^2}.$$

Using this, give the quotient when $x^5 - 23x^2 + 15x + 81$ is divided by $x^2 - 4x + 4$.

54. [slow] Write $\dfrac{x^3 + 4x^2 + x}{x^2 + 4x - 5}$ as the sum of a polynomial and some partial fractions.

55. [very slow] Give the partial fraction decomposition of $\dfrac{x^6 + x^3 + x^2 + 2}{x^3 + x}$.