Linear Algebra, Winter 2021

List 7

Linear combinations, span, matrices

89. Let $\vec{e_1} = 2\vec{\imath}$ and $\vec{e_2} = \vec{\imath} + 3\vec{\jmath}$.
    (a) Find values of a and b such that $3\,\vec{\imath} + 3\,\vec{\jmath} = a\vec{e_1} + b\vec{e_2}$. $\boxed{a = 1, b = 1}$
    (b) Find values of a and b such that $-12\,\vec{\jmath} = a\vec{e_1} + b\vec{e_2}$. $\boxed{a = 2, b = -4}$
    (c) Find values of a and b such that $5\,\vec{\imath} + \vec{\jmath} = a\vec{e_1} + b\vec{e_2}$. $\boxed{a = \frac{7}{3}, b = \frac{1}{3}}$

90. Is it possible to write $\langle 8, 3, 1\rangle$ as a linear combination of $\langle 1, 0, 1\rangle$ and $\langle 2, 0, 3\rangle$?
    $\boxed{\text{No}}$

91. Is it possible to write $\langle 5, 5, 1\rangle$ as a linear combination of $\langle 1, 1, 1\rangle$ and $\langle 0, 0, 8\rangle$?
    $\boxed{\text{Yes}}$

$$\langle 5, 5, 1\rangle = 5\langle 1, 1, 1\rangle + (-\tfrac{1}{2})\langle 0, 0, 8\rangle$$

92. (a) Is $\langle 5, 5, 1\rangle$ is the span of $\langle 1, 1, 1\rangle$ and $\langle 0, 0, 8\rangle$? $\boxed{\text{Yes}}$
    (b) Is $\langle 1, 1, 1\rangle$ is the span of $\langle 5, 5, 1\rangle$ and $\langle 0, 0, 8\rangle$? $\boxed{\text{Yes}}$
    (c) Is $\langle 0, 0, 8\rangle$ is the span of $\langle 1, 1, 1\rangle$ and $\langle 5, 5, 1\rangle$? $\boxed{\text{Yes}}$

93. Describe in words the span of $\vec{a} = \langle 1, 3, 5\rangle$ and $\vec{b} = \langle -2, 0, 4\rangle$.
    $\boxed{\text{All linear combinations of } \vec{a} \text{ and } \vec{b}.}$

94. Give the dimensions of the following matrices:
    (a) $\begin{bmatrix} -92 & 8 \\ -78 & -67 \end{bmatrix}$ $\boxed{2 \times 2}$
    (b) $\begin{bmatrix} -36 \\ 72 \\ -12 \end{bmatrix}$ $\boxed{3 \times 1}$
    (c) $\begin{bmatrix} 75 & 89 & 50 \\ -5 & -81 & 34 \end{bmatrix}$ $\boxed{2 \times 3}$
    (d) $\begin{bmatrix} -13 & -63 & -5 \\ 0 & -66 & \frac{1}{2} \\ 31 & \frac{5}{22} & \frac{8}{11} \end{bmatrix}$ $\boxed{3 \times 3}$
    (e) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\boxed{2 \times 2}$
    (f) $\begin{bmatrix} 58 & -65 & 40 & -72 & 8 & -1 & 26 \\ -74 & 58 & -92 & 75 & -4 & -21 & 74 \end{bmatrix}$ $\boxed{2 \times 7}$

95. If A is a 2×2 matrix, B is a 3×3 matrix, and C is a 3×2 matrix, which of the following exist?

(a) AA exists (2×2)
(b) AB doesn't exist
(c) AC doesn't exist
(d) BA doesn't exist
(e) BB exists (3×3)
(f) BC exists (3×2)
(g) CA exists (3×2)
(h) CB doesn't exist
(i) CC doesn't exist
(j) ABC doesn't exist
(k) BCA exists (3×2)
(ℓ) ACA doesn't exist

96. Compute the following:

(a) $\begin{bmatrix} 1 & 14 & 8 \\ 6 & -1 & 14 \\ 5 & 11 & -3 \end{bmatrix} + \begin{bmatrix} 11 & 2 & 10 \\ -2 & 14 & 8 \\ -2 & 3 & -4 \end{bmatrix} = \boxed{\begin{bmatrix} 12 & 16 & 18 \\ 4 & 13 & 22 \\ 3 & 14 & -7 \end{bmatrix}}$

(b) $\begin{bmatrix} 1 & 14 & 8 \\ 6 & -1 & 14 \\ 5 & 11 & -3 \end{bmatrix} - \begin{bmatrix} 11 & 2 & 10 \\ -2 & 14 & 8 \\ -2 & 3 & -4 \end{bmatrix} = \boxed{\begin{bmatrix} -10 & 12 & -2 \\ 8 & -15 & 6 \\ 7 & 8 & 1 \end{bmatrix}}$

(c) $3 \begin{bmatrix} 0 & -4 & 0 \\ -1 & -1 & 3 \\ -2 & 5 & 14 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -12 & 0 \\ -3 & -3 & 9 \\ -6 & 15 & 42 \end{bmatrix}}$

(d) $\frac{1}{6} \begin{bmatrix} 9 & 14 \\ 6 & 10 \end{bmatrix} = \boxed{\begin{bmatrix} 3/2 & 7/3 \\ 1 & 5/3 \end{bmatrix}}$

(e) $\begin{bmatrix} 8 & 5 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 33 \\ -25 \end{bmatrix}}$

(f) $\begin{bmatrix} 9 & -2 \\ 8 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \boxed{\begin{bmatrix} 6 \\ 46 \end{bmatrix}}$

(g) $\begin{bmatrix} -5 & 5 & 7 \\ -2 & -3 & 2 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 8 \end{bmatrix} = \boxed{\begin{bmatrix} 86 \\ -17 \\ 35 \end{bmatrix}}$

(h) $\begin{bmatrix} 4 & 8 & 0 \\ -3 & 3 & -3 \\ 8 & 5 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix} = \boxed{\begin{bmatrix} 24 \\ -3 \\ -10 \end{bmatrix}}$

97. Compute $\begin{bmatrix} 1 & -\sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & \sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$. $= \boxed{\begin{bmatrix} 2-\sqrt{2} \\ 6\sqrt{2} \\ 2+\sqrt{2} \end{bmatrix}}$

98. Compute the following, if they exist:

(a) $\begin{bmatrix} 9 & -4 \\ -5 & -5 \end{bmatrix} \begin{bmatrix} 8 & 1 \\ 0 & -3 \end{bmatrix} = \boxed{\begin{bmatrix} 72 & 21 \\ -40 & 10 \end{bmatrix}}$

(b) $\begin{bmatrix} 4 & 5 & 22 \\ 8 & -13 & 4 \end{bmatrix} \begin{bmatrix} 19 & 0 & 35 & 6 \\ 0 & 2 & 2 & 6 \\ 9 & 1 & 19 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} 274 & 32 & 568 & 32 \\ 188 & -22 & 330 & -34 \end{bmatrix}}$

(c) $\begin{bmatrix} 19 & 0 & 35 & 6 \\ 0 & 2 & 2 & 6 \\ 9 & 1 & 19 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 22 \\ 8 & -13 & 4 \end{bmatrix}$ $\boxed{\text{does not exist}}$

(d) $\begin{bmatrix} 3 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 & 2 & 7 \\ 3 & -4 & -1 & 1 & 8 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 6 & -3 & 6 & 21 \\ 6 & -4 & -4 & 6 & 30 \end{bmatrix}}$

(e) $\begin{bmatrix} -2 & -4 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 2 & 8 \end{bmatrix} = \boxed{\begin{bmatrix} 72 & 48 \\ -36 & -24 \end{bmatrix}}$

(f) $\begin{bmatrix} -4 & -3 & -5 \\ 24 & 6 & 29 \end{bmatrix} \begin{bmatrix} 4 & 13 & 0 \\ 2 & -26 & 9 \end{bmatrix}$ $\boxed{\text{does not exist}}$

99. Compute the following:

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & 2 \\ 3 & -3 \end{bmatrix} = \boxed{\begin{bmatrix} 8 & 2 \\ 3 & -3 \end{bmatrix}}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 14 & 21 \\ -11 & 23 \end{bmatrix} = \boxed{\begin{bmatrix} 14 & 21 \\ -11 & 23 \end{bmatrix}}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 99 & \frac{1}{10} \\ -37 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 99 & \frac{1}{10} \\ -37 & 2 \end{bmatrix}}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -2 & 1 \\ 5 & 2 & 5 \\ 7 & 4 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -4 & -2 & 1 \\ 5 & 2 & 5 \\ 7 & 4 & 1 \end{bmatrix}}$

(e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 15 & 11 & -15 \\ 17 & 10 & -8 \\ 0 & 0 & -13 \end{bmatrix} = \boxed{\begin{bmatrix} 15 & 11 & -15 \\ 17 & 10 & -8 \\ 0 & 0 & -13 \end{bmatrix}}$

(f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 59 & 28 & 58 \\ 61 & 44 & 67 \\ 22 & 39 & 17 \end{bmatrix} = \boxed{\begin{bmatrix} 59 & 28 & 58 \\ 61 & 44 & 67 \\ 22 & 39 & 17 \end{bmatrix}}$

(g) $\begin{bmatrix} 59 & 28 & 58 \\ 61 & 44 & 67 \\ 22 & 39 & 17 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 59 & 28 & 58 \\ 61 & 44 & 67 \\ 22 & 39 & 17 \end{bmatrix}}$

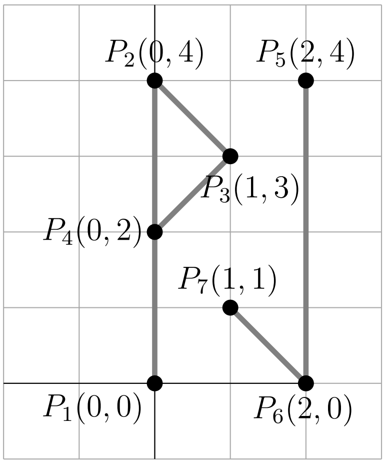
(h) $\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}-39&-66&84&66&-10\\-47&-5&17&-59&-3\\-94&-90&-5&86&31\\25&80&0&35&19\\-72&40&99&48&57\end{bmatrix} = \begin{bmatrix}-39&-66&84&66&-10\\-47&-5&17&-59&-3\\-94&-90&-5&86&31\\25&80&0&35&19\\-72&40&99&48&57\end{bmatrix}$

100. For each of the points P_1 through P_7, calculate

$$P_i' = \begin{bmatrix}1 & 1/2\\0 & 1\end{bmatrix} P_i.$$

(For example, for $P_5' = [\begin{smallmatrix}1 & 1/2\\0 & 1\end{smallmatrix}][\begin{smallmatrix}2\\4\end{smallmatrix}] = [\begin{smallmatrix}4\\4\end{smallmatrix}]$.) Plot the points $P_1', \ldots, P_7'$ on a new grid. Connect $P_1' \to P_2' \to P_3' \to P_4'$ with line segments, and connect $P_5' \to P_6' \to P_7'$.

Congratulations. You can write italic.



$T(P_1) = (0,0)$ $T(P_2) = (2,4)$ $T(P_3) = (5/2,3)$ $T(P_4) = (1,2)$
$T(P_5) = (4,4)$ $T(P_6) = (2,0)$ $T(P_7) = (3/2,1)$