

**List 7**

*Linear combinations, span, matrices*

89. Let  $e_1 = 2\vec{i}$  and  $e_2 = \vec{i} + 3\vec{j}$ .
- Find values of  $a$  and  $b$  such that  $3\vec{i} + 3\vec{j} = ae_1 + be_2$ .
  - Find values of  $a$  and  $b$  such that  $-12\vec{j} = ae_1 + be_2$ .
  - Find values of  $a$  and  $b$  such that  $5\vec{i} + \vec{j} = ae_1 + be_2$ .
90. Is it possible to write  $\langle 8, 3, 1 \rangle$  as a linear combination of  $\langle 1, 0, 1 \rangle$  and  $\langle 2, 0, 3 \rangle$ ?
91. Is it possible to write  $\langle 5, 5, 1 \rangle$  as a linear combination of  $\langle 1, 1, 1 \rangle$  and  $\langle 0, 0, 8 \rangle$ ?
92. (a) Is  $\langle 5, 5, 1 \rangle$  in the span of  $\langle 1, 1, 1 \rangle$  and  $\langle 0, 0, 8 \rangle$ ?  
 (b) Is  $\langle 1, 1, 1 \rangle$  in the span of  $\langle 5, 5, 1 \rangle$  and  $\langle 0, 0, 8 \rangle$ ?  
 (c) Is  $\langle 0, 0, 8 \rangle$  in the span of  $\langle 1, 1, 1 \rangle$  and  $\langle 5, 5, 1 \rangle$ ?
93. Describe in words the span of  $\vec{a} = \langle 1, 3, 5 \rangle$  and  $\vec{b} = \langle -2, 0, 4 \rangle$ .
94. Give the dimensions of the following matrices:
- $\begin{bmatrix} -92 & 8 \\ -78 & -67 \end{bmatrix}$
  - $\begin{bmatrix} -36 \\ 72 \\ -12 \end{bmatrix}$
  - $\begin{bmatrix} 75 & 89 & 50 \\ -5 & -81 & 34 \end{bmatrix}$
  - $\begin{bmatrix} -13 & -63 & -5 \\ 0 & -66 & \frac{1}{2} \\ 31 & \frac{5}{22} & \frac{8}{11} \end{bmatrix}$
  - $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
  - $\begin{bmatrix} 58 & -65 & 40 & -72 & 8 & -1 & 26 \\ -74 & 58 & -92 & 75 & -4 & -21 & 74 \end{bmatrix}$
95. If  $A$  is a  $2 \times 2$  matrix,  $B$  is a  $3 \times 3$  matrix, and  $C$  is a  $3 \times 2$  matrix, which of the following exist?
- $AA$
  - $AB$
  - $AC$
  - $BA$
  - $BB$
  - $BC$
  - $CA$
  - $CB$
  - $CC$
  - $ABC$
  - $BCA$
  - $ACA$

96. Compute the following:

- $\begin{bmatrix} 1 & 14 & 8 \\ 6 & -1 & 14 \\ 5 & 11 & -3 \end{bmatrix} + \begin{bmatrix} 11 & 2 & 10 \\ -2 & 14 & 8 \\ -2 & 3 & -4 \end{bmatrix}$
- $\begin{bmatrix} 1 & 14 & 8 \\ 6 & -1 & 14 \\ 5 & 11 & -3 \end{bmatrix} - \begin{bmatrix} 11 & 2 & 10 \\ -2 & 14 & 8 \\ -2 & 3 & -4 \end{bmatrix}$
- $3 \begin{bmatrix} 0 & -4 & 0 \\ -1 & -1 & 3 \\ -2 & 5 & 14 \end{bmatrix}$
- $\frac{1}{6} \begin{bmatrix} 9 & 14 \\ 6 & 10 \end{bmatrix}$

$$(e) \begin{bmatrix} 8 & 5 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$(g) \begin{bmatrix} -5 & 5 & 7 \\ -2 & -3 & 2 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 8 \end{bmatrix}$$

$$(f) \begin{bmatrix} 9 & -2 \\ 8 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$(h) \begin{bmatrix} 4 & 8 & 0 \\ -3 & 3 & -3 \\ 8 & 5 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}$$

97. Compute  $\begin{bmatrix} 1 & -\sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & \sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$ .

98. Compute the following, if they exist:

$$(a) \begin{bmatrix} 9 & -4 \\ -5 & -5 \end{bmatrix} \begin{bmatrix} 8 & 1 \\ 0 & -3 \end{bmatrix}$$

$$(d) \begin{bmatrix} 3 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 & 2 & 7 \\ 3 & -4 & -1 & 1 & 8 \end{bmatrix}$$

$$(b) \begin{bmatrix} 4 & 5 & 22 \\ 8 & -13 & 4 \end{bmatrix} \begin{bmatrix} 19 & 0 & 35 & 6 \\ 0 & 2 & 2 & 6 \\ 9 & 1 & 19 & -1 \end{bmatrix}$$

$$(e) \begin{bmatrix} -2 & -4 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 2 & 8 \end{bmatrix}$$

$$(c) \begin{bmatrix} 19 & 0 & 35 & 6 \\ 0 & 2 & 2 & 6 \\ 9 & 1 & 19 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 22 \\ 8 & -13 & 4 \end{bmatrix}$$

$$(f) \begin{bmatrix} -4 & -3 & -5 \\ 24 & 6 & 29 \end{bmatrix} \begin{bmatrix} 4 & 13 & 0 \\ 2 & -26 & 9 \end{bmatrix}$$

99. Compute the following:

$$(a) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & 2 \\ 3 & -3 \end{bmatrix}$$

$$(f) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 59 & 28 & 58 \\ 61 & 44 & 67 \\ 22 & 39 & 17 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 14 & 21 \\ -11 & 23 \end{bmatrix}$$

$$(g) \begin{bmatrix} 59 & 28 & 58 \\ 61 & 44 & 67 \\ 22 & 39 & 17 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 99 & \frac{1}{10} \\ -37 & 2 \end{bmatrix}$$

$$(h) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -39 & -66 & 84 & 66 & -10 \\ -47 & -5 & 17 & -59 & -3 \\ -94 & -90 & -5 & 86 & 31 \\ 25 & 80 & 0 & 35 & 19 \\ -72 & 40 & 99 & 48 & 57 \end{bmatrix}$$

$$(d) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -2 & 1 \\ 5 & 2 & 5 \\ 7 & 4 & 1 \end{bmatrix}$$

$$(e) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 15 & 11 & -15 \\ 17 & 10 & -8 \\ 0 & 0 & -13 \end{bmatrix}$$

100. For each of the points  $P_1$  through  $P_7$ , calculate

$$P'_i = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} P_i.$$

(For example, for  $P'_5 = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ .) Plot the points  $P'_1, \dots, P'_7$  on a new grid. Connect  $P'_1 \rightarrow P'_2 \rightarrow P'_3 \rightarrow P'_4$  with line segments, and connect  $P'_5 \rightarrow P'_6 \rightarrow P'_7$ .

*Congratulations. You can write italic.*

