Math 01124
Reasoning with Functions

Thursday, September 5

Instructor: Adam Abrams

Section 17
CRN (Course Registration Number) 45656
Tuesday/Thursday/Friday 3:30 - 4:45 pm

Section 18
CRN (Course Registration Number) 45703
Monday/Wednesday/Thursday 9:30 - 10:45 am

Course policies

Last time

We will use **Canvas** later, but for now course materials are at

theadamabrams.com/01124

Grades come from six quizzes (one dropped), one final exam, and participation. Total possible $15 \times 5 + 20 + 5 = 100$.

- For each quiz, 2 points can be earned from homework (still max 15/15).

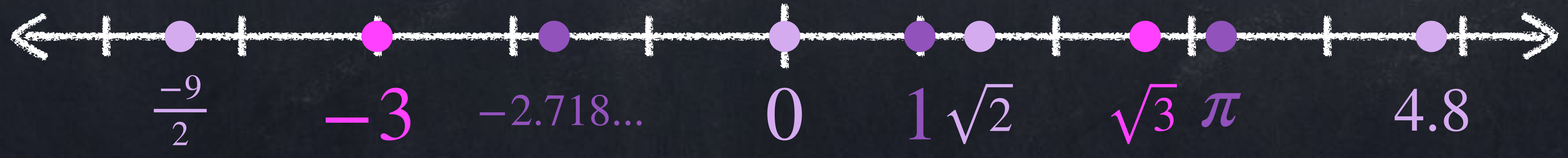
Office of Accessibility: sites.rowan.edu/accessibilityservices/

Calculators: allowed, but not required

Last time

Types of numbers

- **Natural** numbers: $0, 1, 2, 3, 4, \ldots$ (in some books, only $1, 2, 3, 4, \ldots$)
- **Integers**: $\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$
- **Rational** numbers are all the numbers that *can* be written as one integer divided by another integer. Examples: $\frac{1}{2}, \frac{-2}{3}, 1.5, \frac{8}{1} = 8, 0, \frac{-5}{4}$
- **Real** numbers are all the values on a number line. Examples:



- Real numbers that are *not* rational are called **irrational**.

$+ - \times \div$ with fractions

Multiplying fractions is extremely easy: $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$.

To add fractions, first make sure they have the same denominators:

$$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}.$$

Subtracting fractions is very similar to addition: $\frac{1}{2} - \frac{3}{4} = \frac{2}{4} - \frac{3}{4} = \frac{-1}{4}$.

Dividing by x is the same as multiplying by $\frac{1}{x}$, and $\frac{1}{\frac{a}{b}} = \frac{b}{a}$.

+ − × ÷ with fractions

Dividing by x is the same as multiplying by $\frac{1}{x}$, and $\frac{1}{\frac{a}{b}} = \frac{b}{a}$.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6}$$, which simplifies to $\frac{2}{3}$.

We will not use the ÷ symbol often. This would more likely be written as

$$\frac{1/2}{3/4} = \frac{4}{6} = \frac{2}{3}.$$

1. Expand

$$(4t - s) + (\frac{2}{3}t + 6s),$$

(that is, write it in a format that does not use parentheses) and then simplify.

2. Expand $(4t - s) - (\frac{2}{3}t + 6s)$.

3. Using the distributive property ($a(b + c) = ab + ac$), expand

$$(4t - s)(\frac{2}{3}t + 6s).$$

Properties of exponents

- $a^n \cdot a^m = a^{n+m}$
- $(a^n)^m = a^{nm}$
- $(a \cdot b)^n = a^n \cdot b^n$ ← "distributive property for powers over ×"

Task: Simplify $5(-2a^3b^{-1})(\frac{b}{a})^2$.

Question: What is the value of $9^{1/2} \times 9^{1/2}$?

Square roots

For $x \geq 0$, the number $\sqrt{x}$ (spoken as "the square root of x", or just "root x") is the non-negative number for which $\sqrt{x} \cdot \sqrt{x} = x$.

- For "perfect squares", the square root is a natural number: $\sqrt{36} = 6$.
- For some, there is no simpler way to write them: $\sqrt{5}$ is just $\sqrt{5}$.
- For some, we can simplify or re-write them a little bit but will still have to use √ somewhere.

Important property: if $a, b > 0$ then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Square roots

- For some, we can simplify or re-write them a little bit but will still have to use √ somewhere.

Important property: if $a, b > 0$ then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example: $\sqrt{20}$ = ?

Example: Simplify $\dfrac{\sqrt{234x^{11}y}}{\sqrt{26x^7y}}$.

+/− with √

Usually, *sums* of square roots can't be simplified.

- $\sqrt{2} \cdot \sqrt{3}$ is also $\sqrt{6}$.
- $\sqrt{2} + \sqrt{3}$ is… just $\sqrt{2} + \sqrt{3}$.

However, you can simplify if the same square root appears multiple times.

- Example 1: Simplify $\sqrt{2} - 8\sqrt{2}$.
- Example 2: Simplify $5\sqrt{12} + 2\sqrt{3}$.

Other roots

We have seen that $x^{1/2} = \sqrt{x}$.

In general, $x^{1/n} = \sqrt[n]{x}$, which is the number for which $(?)^n = x$.

We also know $(x^a)^b = x^{ab}$, so we can combine these ideas to get

$$x^{m/n} = \sqrt[n]{x^m}.$$

Polynomials

A **polynomial** formula with the variable x looks like

😀$x^n +$ 🧐$x^{n-1} + \cdots +$ 😂$x^2 +$ 🥺$x +$ 😐,

where $n \geq 0$ is an integer, and the faces are numbers (or variables) that we call **coefficients**.

- Each 😶x^k is called a **term** of the polynomial.
- The term that does not include x at all is the **constant term**.
- The term with the highest power of x is called the **leading term**.
  - The exponent in the leading term is the **degree** of the polynomial.